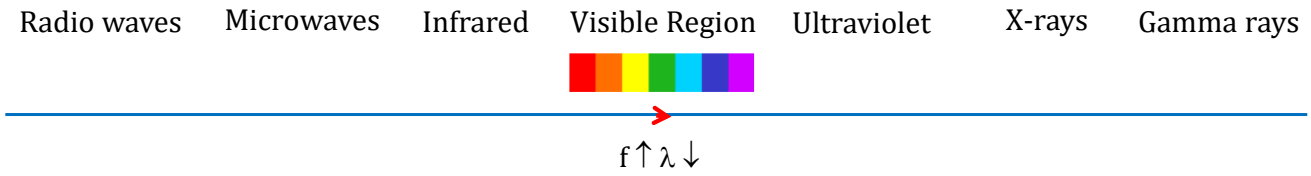
# 01 Ray Optics and Optical Instruments

## Introduction

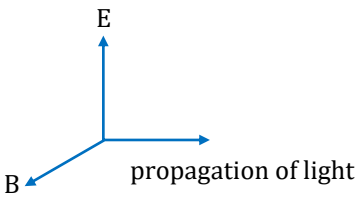
**Optics:** Optics is the branch of physics in which we study the behaviour and properties of light, including its interactions with matter.

Light is that part of electromagnetic waves which gives sensation to our eyes.




## Properties of light

1. Speed of light in vacuum, denoted by $c = 3 \times 10^8$ m/s approximately.
2. Light carries energy and momentum.
3. Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric and magnetic field.
4. Light is an electromagnetic wave and transverse in nature.
5. The formula $v = f\lambda$ is applicable to light.




**Note:** Eye is most sensitive for yellowish green colour and least sensitive for violet and red colours.

## Rectilinear propagation of light

Rectilinear propagation describes the tendency of electromagnetic waves (light) to travel in a straight line path in homogenous medium.

## Ray

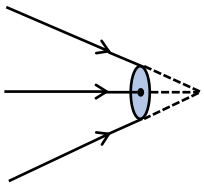
A **ray** of light is the straight-line path of transfer of light energy. Arrow represents the direction of propagation of light.

Figure shows a ray which indicates light is moving from A to B.

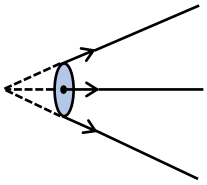
## Beam

A bundle or bunch of rays is called a beam. It is of following three types :

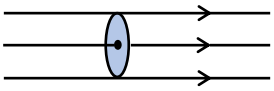
**(a)** **Convergent beam :** In this case diameter of beam decreases in the direction of ray.



**(b)** **Divergent beam :** It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.



**(c)** **Parallel beam :** It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same.



Parallel beam of light

**OPTICS**

**Ray Optics (Geometrical Optics)**

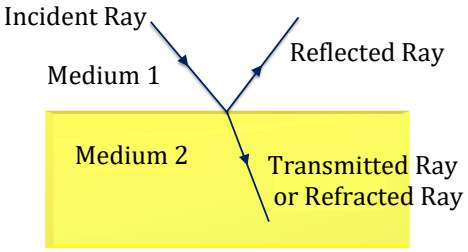
Study of light in the form of ray

**Wave Optics**

Study of light in the form of wave

Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomenon is known as geometrical optics.

## Reflection of Light




When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

### Laws of Reflection

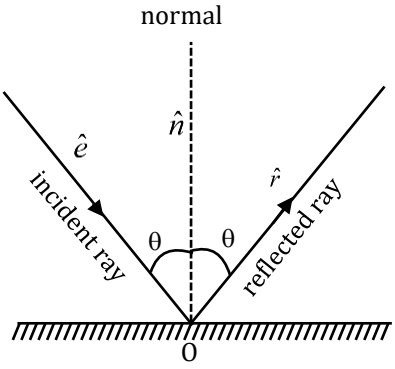
(1) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**.

This condition can be expressed mathematically as $\hat{e}.(\hat{n}\times\hat{r})=0$

Where; $\hat{e}$ is a unit vector along incident ray.

$\hat{n}$ is a unit vector along normal.

$\hat{r}$ is a unit vector along reflected ray.



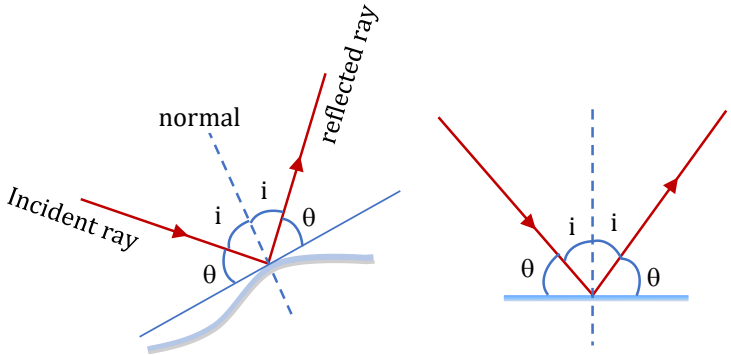

(2) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e. $\angle i = \angle r$

### Note:

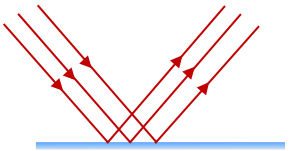
**(a)** The frequency, wavelength and speed of light do not change after reflection.

**(b)** If we draw a tangent at the point of incident then the angle between incident ray and tangent is equal to the angle between reflected ray and tangent.

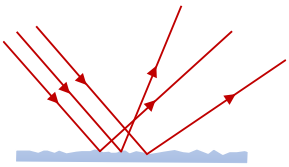
In case of plane mirror angle between incident ray and plane mirror is same as angle between reflected ray and plane mirror.



## Types of Reflection



1. **Regular Reflection:** When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.
   **Eg :-** reflection from mirror.



2. **Diffused Reflection:** When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface, and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as **diffused reflection**.
   **Eg :-** reflection from a wall, newspaper etc.

## Object and Image

**Object:** Light rays incident on an optical element is responsible to define object.
Object is defined as the point of intersection of incident rays.

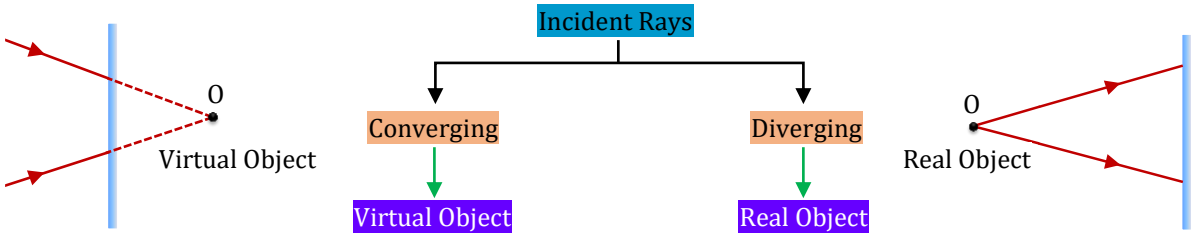
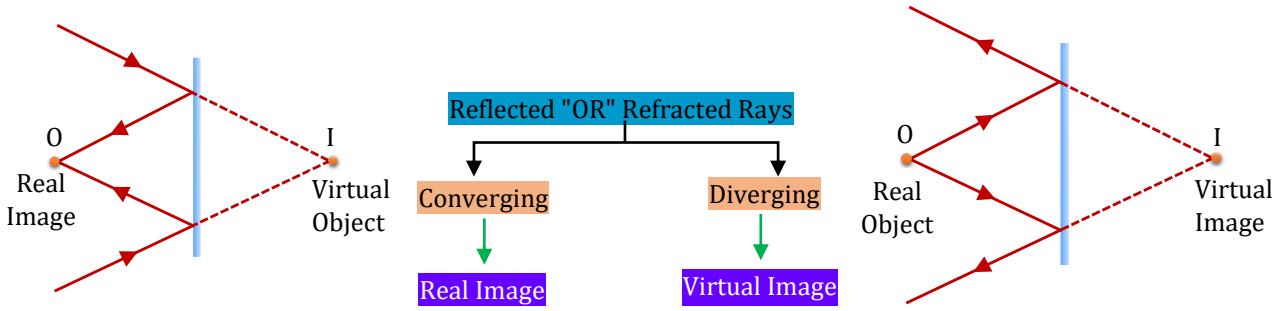


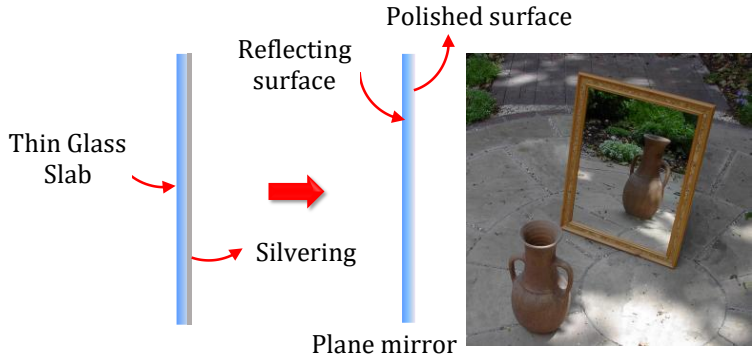

**Image:** Light rays formed after interaction (either reflection or refraction) with an optical element is responsible to define image.
Image is defined as the point of intersection of reflected or refracted rays.




**Note:** A real image can be captured on a screen. This image on the screen can be perceived by the eye.
What about virtual images? Since they cannot be captured on a screen is it possible to see them.
From practical experience we know that we can see ourselves in a plane mirror which is indeed a virtual image. So, it is possible to see a virtual image. But how is it done?
There is a lens in our eye that focuses the diverging set of rays on to the retina forming a real image inside our eyes.
In a similar manner, a camera can also convert a virtual image into a real image which is captured on the film.

## Reflection from Plane Mirror

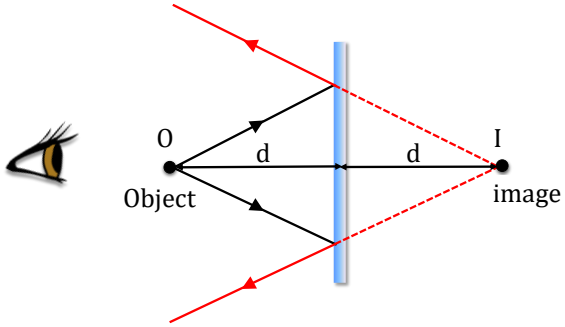
A plane mirror is a mirror with a flat (planar) reflective surface.




A plane mirror behaves like a window to a virtual world.

### Image formation in plane mirror

Diverging light rays leave the source (object) and are reflected from the mirror, obeying the law of reflection. Upon reflection, the rays continue to diverge. The dashed lines in figure are backward extensions of the diverging rays back to a point of intersection at I. The diverging rays appear to originate at the point I behind the mirror to the observer. Point I, which is behind the mirror, is called the **image** of the object at O.




**Perpendicular distance of object from plane mirror is equal to perpendicular distance of image from plane mirror.**

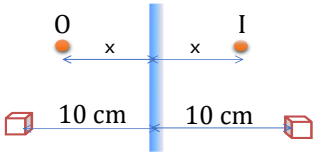
**Characteristics of image due to reflection by a plane mirror:**

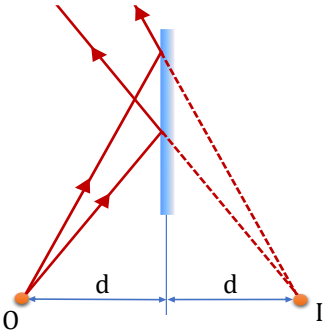
**(i)** Distance of object from mirror = Distance of image from the mirror.

**(ii)** All the incident rays from a point object will meet or appear to meet at a single point after reflection from a plane mirror which is called image.

**(iii)** The line joining a point object and its image is normal to the reflecting surface.

**For example:**



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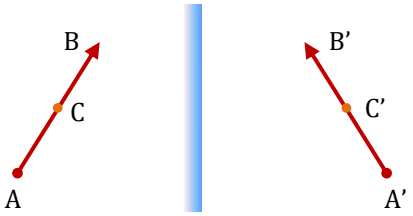
## Extended object

An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A′ will be image of A, C′ will be image of C, B′ will be image of B etc. All point images together form extended image. Thus, extended image is formed of an extended object.

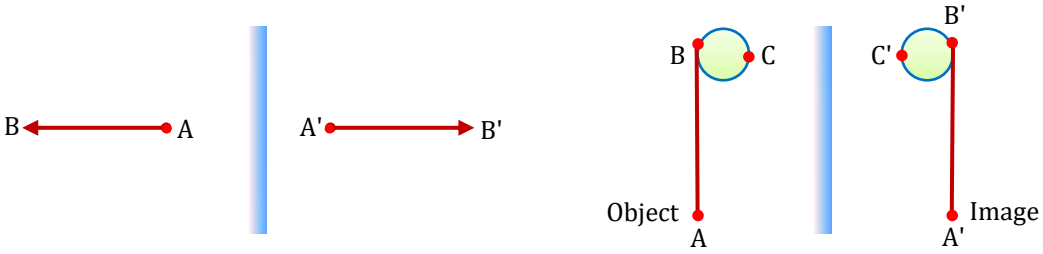



## Properties of image of an extended object, formed by a plane mirror

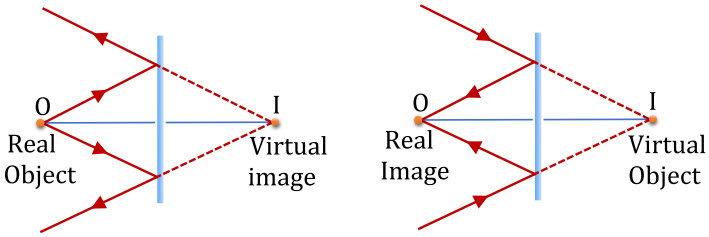
(i) Size of extended object = Size of extended image.

(ii) The image is virtual & erect, if the real extended object is placed parallel to the mirror.

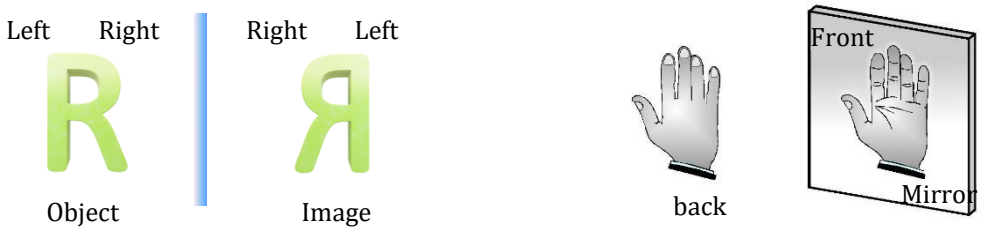
(iii) The image is inverted if the extended object lies perpendicular to the plane mirror.




(iv) For a real object the image is virtual and for a virtual object the image is real.




**Note:** The image formed by a plane mirror suffers lateral–inversion, i.e., left is turned into right and vice–versa with respect to object in the image formed by a plane mirror.




Some capital English alphabets do not show lateral inversion.

**Example :** A, H, I, M, O, T, U, V, W, X and Y

## Analog clock and plane mirror

When plane mirror is placed in front of the analog clock then the time observed by the observer in image of clock is different from the actual time.

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| (a) If actual time is in **HH : 00** then time observed in the image of clock is, | (b) If actual time is in **HH : MM** then time observed in the image of clock is, | (c) If actual time is in **HH : MM : SS** then time observed in the image of clock is, |
|---|---|---|
| 12 : 00<br>– HH : 00<br>xx : 00 | 11 : 60<br>– HH : MM<br>xx : yy | 11 : 59 : 60<br>– HH : MM : SS<br>xx : yy : zz |

**Illustration 1:**

An unnumbered wall clock shows time 04 : 25 : 37, where 1st term represents hours, 2nd represents minutes & the last term represents seconds. What time will its image in a plane mirror show?

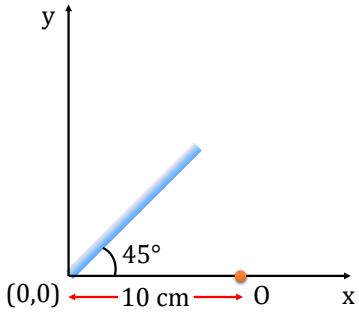
**Solution:**

$$\begin{array}{r} 11:59:60 \\ -\ 04:25:37 \\ \hline 7:34:23 \end{array}$$

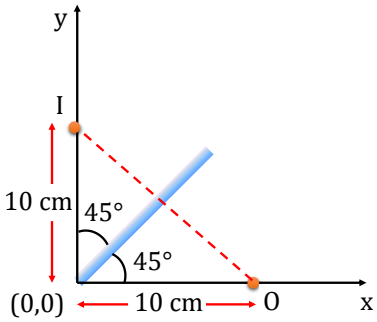
**Note:** The angle that an image makes from the plane mirror is equal to the angle that an object makes from the plane mirror.

**Illustration 2:**

Find out position of image formed after reflection.



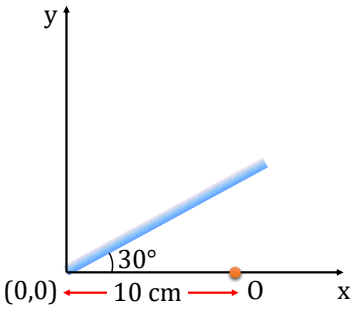

**Solution:**



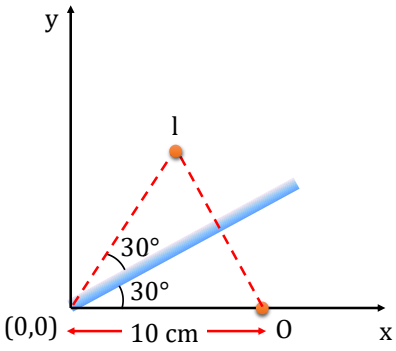

Co-ordinate of image is (0, 10)

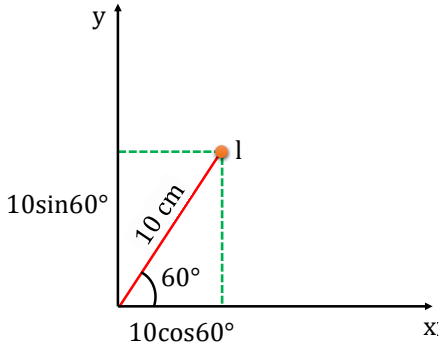
**Illustration 3:**

Find out position of image formed after reflection.



**Solution:**

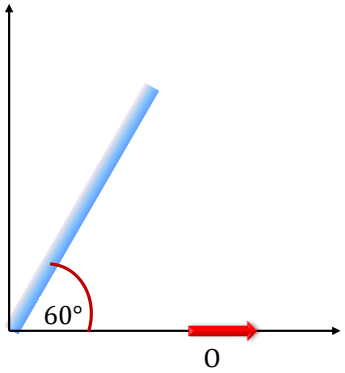


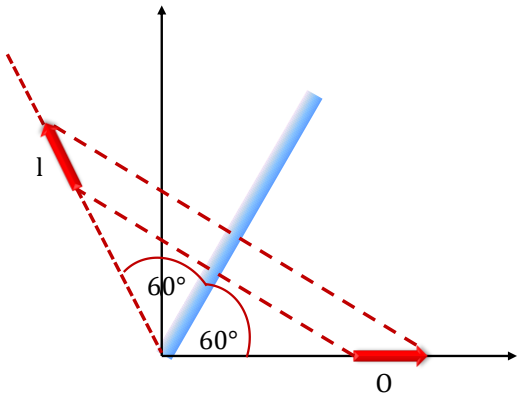

Co-ordinate of image is $(10 \cos 60°, 10 \sin 60°) = (5, 5\sqrt{3})$

**Illustration 4:**

Draw image of an object as shown in figure.




**Solution:**




## Angle of Deviation & Rotation of Plane Mirror

**Angle of deviation (δ) in plane mirror:** When light ray is incident on plane mirror then due to reflection its path get deviated, so the angle by which it got deviated is known as angle of deviation.

### δ v/s *i* curve

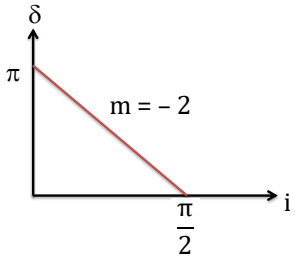
$\delta = \pi - 2i$

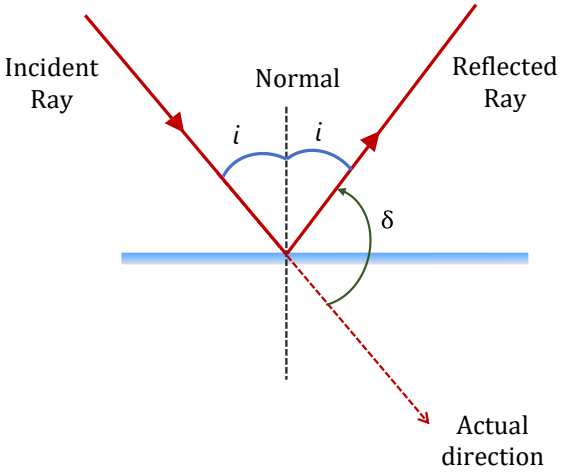
Rearrange the above relation,

$\delta = -2i + \pi$

Compare the above relation with straight line relation $y = mx + c$

Then graph between δ and i is a straight line with negative slope.




$\delta = \pi - 2i$

*or*

$\delta = 180 - 2i$

**Note:** When light strikes the mirror perpendicularly (normally) then after reflection it retrace its path.

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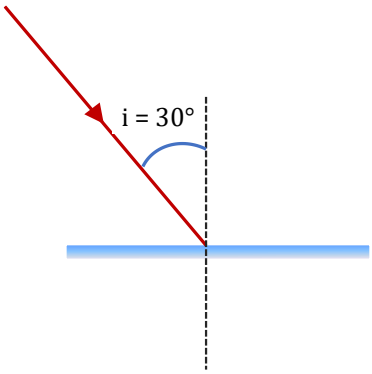
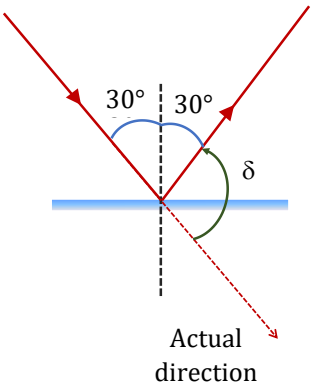
**Illustration 5:**

Find out angle of deviation?

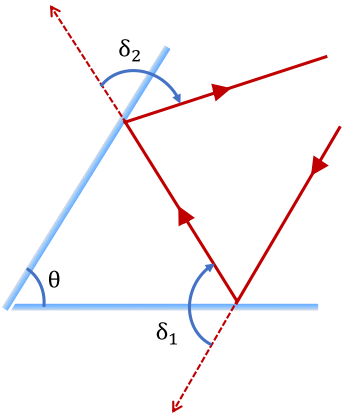



**Solution:**




$\delta = \pi - 2i = 180° - 2\,(30°) = 120°$ (anticlockwise)

**Note:** When reflection are more than one then add angle of deviation of all reflection along with sign.




$\delta_{net} = \delta_1 + \delta_2$

**Illustration 6:**

Find net angle of deviation after two reflection?

**Solution:**

For mirror $M_1$

$\delta_1 = 180° - 2i$

$= 180° - 2(30°)$

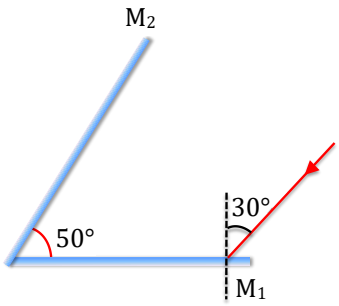
$= 120°$ (clockwise)

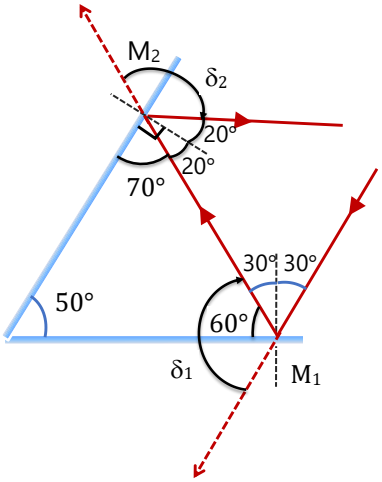
For mirror $M_2$

$\delta_2 = 180° - 2i$

$= 180° - 2(20°)$

$= 140°$ (clockwise)




Total deviation $\delta_{net} = \delta_1 + \delta_2 = 120° + 140° = 260°$ (clockwise)

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**Note:**

For mirror $M_1$, $\delta_1 = 180° - 2\alpha$

For mirror $M_2$, $\delta_2 = 180° - 2\beta$

$\delta_{net} = \delta_1 + \delta_2$

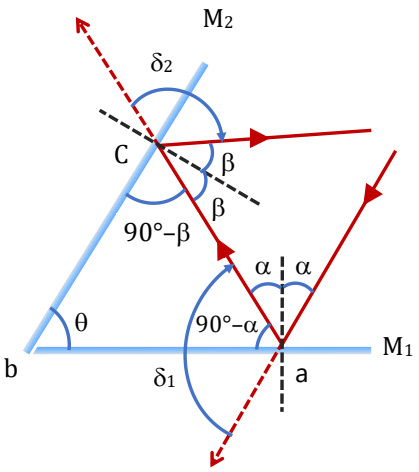
$\delta_{net} = 360° - 2(\alpha + \beta)$

In $\Delta abc$

$90° - \alpha + 90° - \beta + \theta = 180°$

$\alpha + \beta = \theta$

Now, $\boxed{\delta_{net} = 360° - 2\theta}$

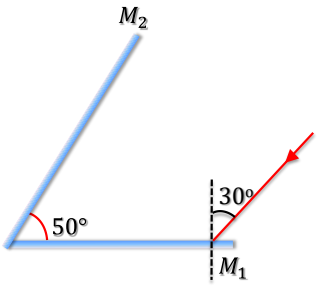



So,

- Total deviation after two reflections, $\delta_{net} = 360° - 2\theta$ (clockwise) ($\theta$ is the angle between both the mirror.)
- $\delta_{net}$ for two reflection is independent of angle of incidence.

**Illustration 7:**

Find net angle of deviation after two reflections?

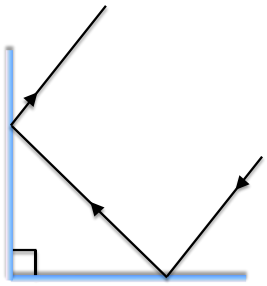



**Solution:**

$\delta_{net} = 360° - 2(50°) = 260°$

**Illustration 8:**

Find net angle of deviation after two reflections?



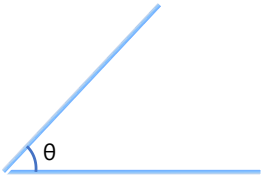
**Solution:**

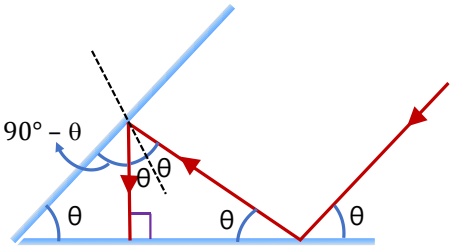
$\delta_{net} = 360° - 2(90) = 180°$

**Note:** When two mirror are perpendicular to each other then reflected ray is antiparallel to incident ray after two reflection.

**Illustration 9:**

Two mirrors are inclined at an angle $\theta$ as shown in the figure. Light ray is incident parallel to one of the mirrors. The ray will start retracing its path after third reflection if –




**Solution:**

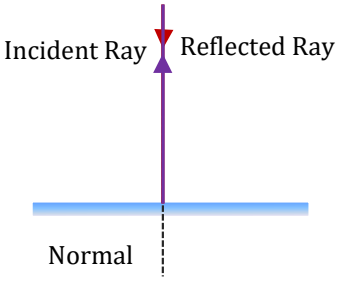
$3\theta + 90° = 180°$

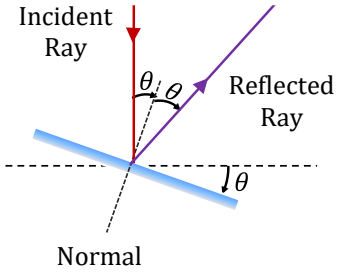
$\theta = 30°$

TG: @Chalnaayaaar

## Rotation of plane mirror

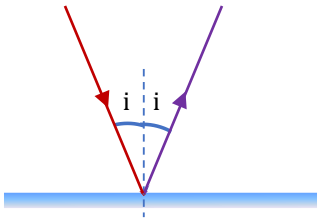
If the direction of the incident ray is kept constant and the mirror is rotated through an angle θ about an axis in the plane of mirror, then the reflected ray rotates through an angle 2θ in the same sense.




## Rotation of Incident Ray

If mirror is kept fixed and incident ray is rotated, then reflected ray will rotate in opposite sense by same angle.



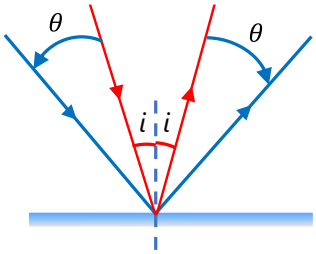
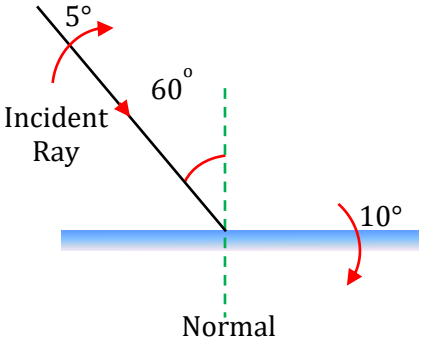




**Illustration 10:**

Figure shows a plane mirror on which a light ray is incident. If the incident light ray is turned by 5° and the mirror by 10°, as shown, find the angle turned by the reflected ray.




**Solution:**

By incident ray: θ = 5° (Anticlockwise)

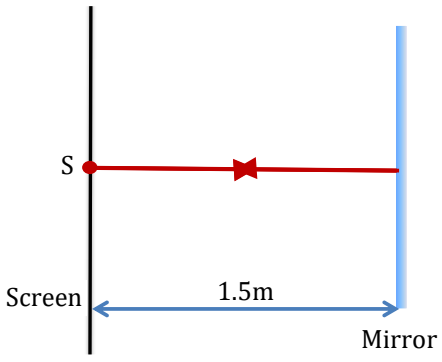
By mirror: 2θ = 2 × 10° = 20° (clockwise)

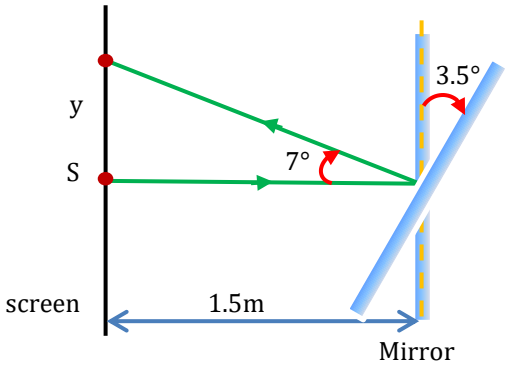
Net rotation = 20° – 5° = 15° (clockwise)

**Illustration 11:**

A ray of light from a source S is incident normally on a plane mirror fixed at a distance 1.5 m from the source. When the mirror is rotated through a small angle 3.5°, the spot of the light is found to move through a distance y on the screen. Find out the value of y.

**Solution:**




$\tan 7° = \frac{y}{1.5}$ (Angle is very small so, $\tan\theta \approx \theta$)

$7° \times \frac{\pi}{180} = \frac{y}{1.5} \Rightarrow y = 0.18$ m

TG: @Chalnaayaaar

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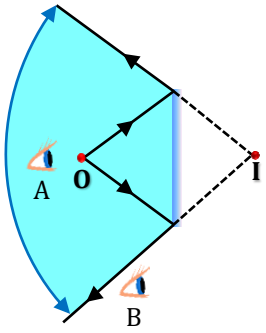
### Field of View

The region in which observer's eye must be present in order to view the image is called field of view.

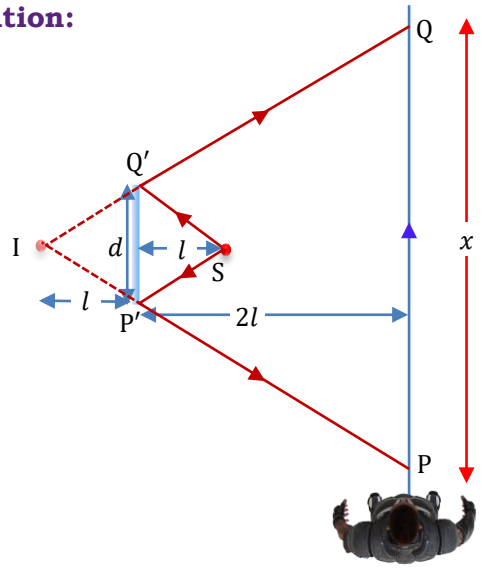
Observer A will able to see the image because he receive reflected rays but observer B can not see the image.




### Illustration 12:

A point source of light S is placed at a distance $l$ in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2l$ from it as shown in the figure. Find the greatest distance over which he can see the image of the light source from the mirror.

**Solution:**




Locate the position of image with the help of ray diagram in which light rays reflect from the ends of the mirror. By the concept of Similar Triangles

$$\Delta P'Q'I \sim \Delta PQI$$

$$\frac{x}{3\ell}=\frac{d}{\ell}\Rightarrow x=3d$$

### Illustration 13:

A boy of height H with his eye-level at height h from the ground stands before a mirror fixed on a wall. Indicate by means of a ray diagram how the mirror should be positioned so that he can view himself fully. What should be the minimum length and position of the mirror?

**Solution:**

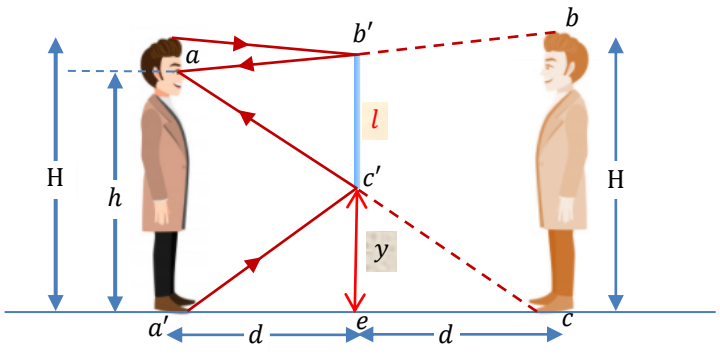
$\Delta ab'c' \sim \Delta abc$

$$\frac{l}{D}=\frac{H}{2d}$$

$$l=\frac{H}{2}$$

$\Delta aa'c \sim \Delta c'ec$

$$\frac{h}{2d}=\frac{y}{d}\Rightarrow y=\frac{h}{2}$$



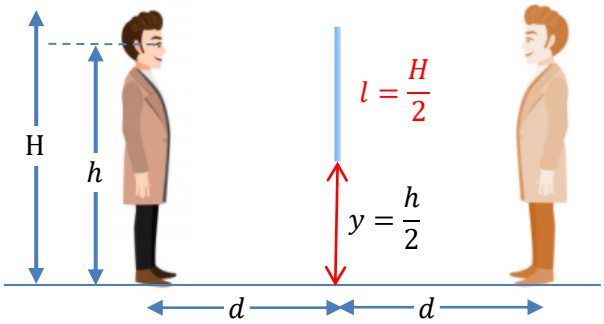

TG: @Chalnaayaaar

## Conclusion

- To see the complete image in a plane mirror the minimum length of plane mirror should be half the height of a person.
- Height of mirror from ground

$$y = \frac{\text{Height of eye level from ground}}{2}$$

- This result does not depend on position of eye (height of the eye from ground).
- This result is independent of the distance of person from the mirror.




**Note:** If height of eyes is not given in question, then consider the eyes at top of the head and solve the question

## Velocity of Image in Plane Mirror

**Case-1:** When object is moving perpendicular to the plane mirror:

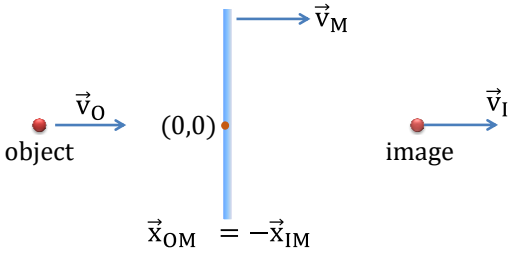
From mirror property: $\vec{x}_{OM} = -\vec{x}_{IM}$

Differentiate with respect to time,

$\vec{v}_{OM} = -\vec{v}_{IM}$

$\vec{v}_O - \vec{v}_M = -(\vec{v}_I - \vec{v}_M)$

$$\boxed{\vec{v}_I = 2\vec{v}_M - \vec{v}_O}$$




$\vec{v}_I$ = velocity of image with respect to ground and perpendicular to the plane of mirror

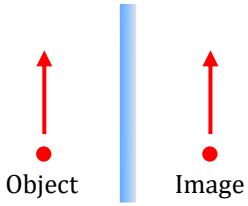
$\vec{v}_o$ = velocity of object with respect to ground and perpendicular to the plane of mirror

$\vec{v}_M$ = velocity of mirror with respect to ground and perpendicular to the plane of mirror

**Case-2:** When object is moving parallel in front of plane mirror:

When object moves parallel to the plane mirror then the velocity of image is same as velocity of object.

$$\boxed{\vec{v}_i = \vec{v}_o}$$

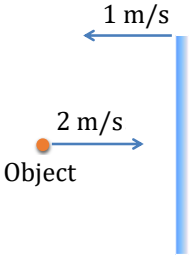



$\vec{v}_i$ = velocity of image with respect to ground and parallel to mirror.

$\vec{v}_o$ = velocity of object with respect to ground and parallel to mirror.

## Illustration 14:

Find velocity of image with respect to ground?




**Solution:**

$V_M = -1$ m/s $\Rightarrow V_o = 2$ m/s

$V_I = 2V_M - V_o \Rightarrow V_I = 2(-1) - 2 = -4$ m/s

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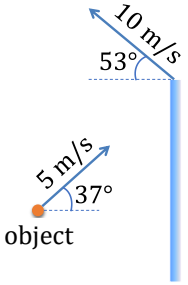
### Illustration 15:

Find velocity of image with respect to ground?




### Solution:

**For perpendicular component**

$V_I = 2V_M - V_o$

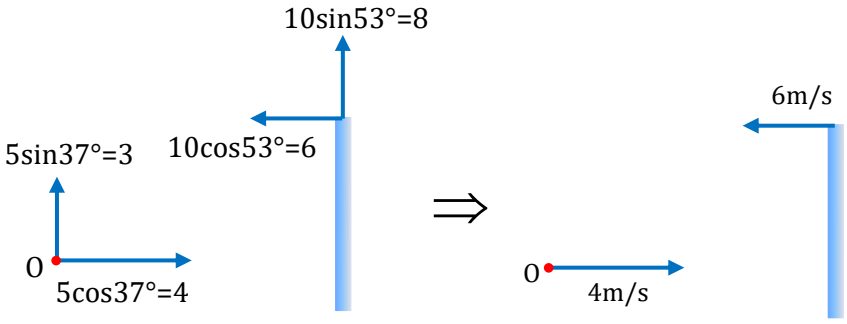
$V_I = 2(-6) - 4 = -16$ m/s

**For parallel component**

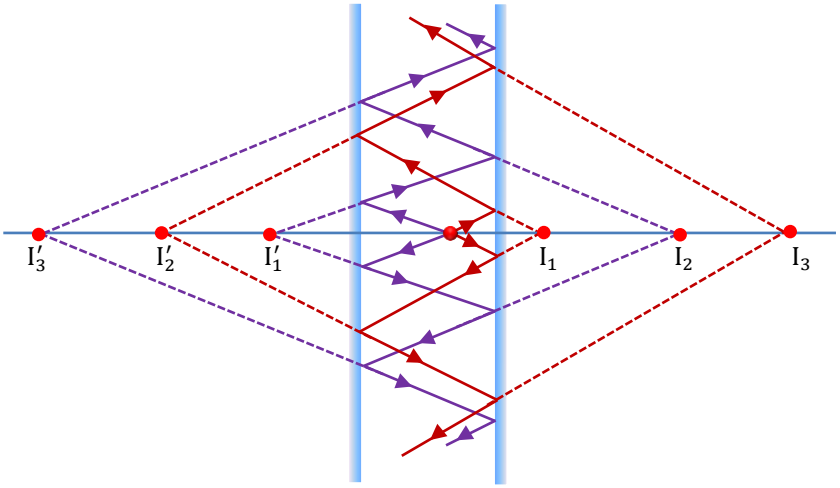
$V_I = V_o = 3$ m/s

So final answer

$\vec{V}_I = -16\hat{i} + 3\hat{j}$




### Multiple Reflections (Number of Images)

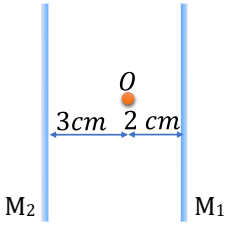



If rays after getting reflected from one mirror strike second mirror, the images formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

For example: Image $I_1$ will act as an object for image $I_2'$ and $I_2'$ will act as an object for image $I_3$.

### Illustration 16:

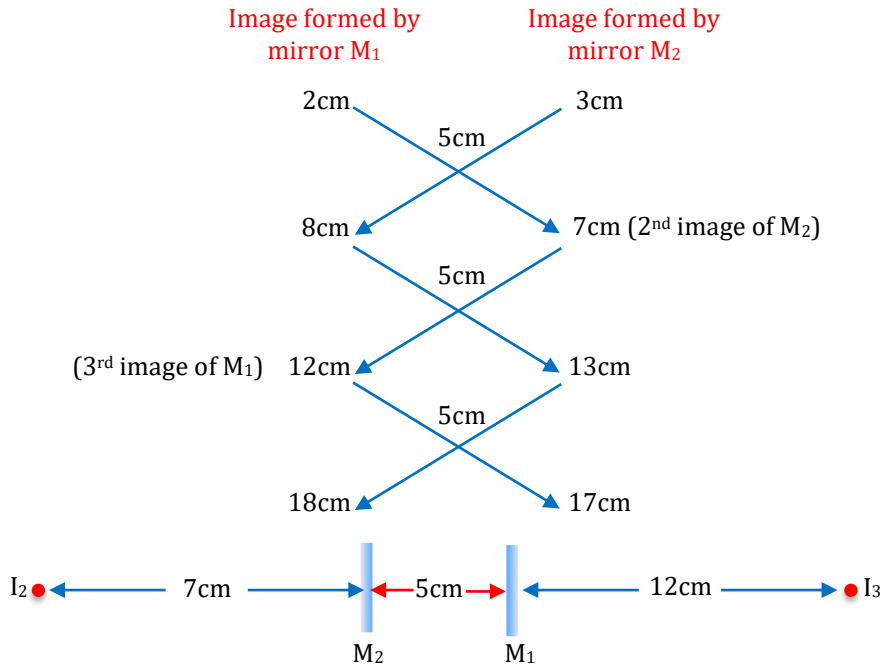
Find out distance between 3rd image of $M_1$ and 2nd image of $M_2$.




TG: @Chalnaayaaar

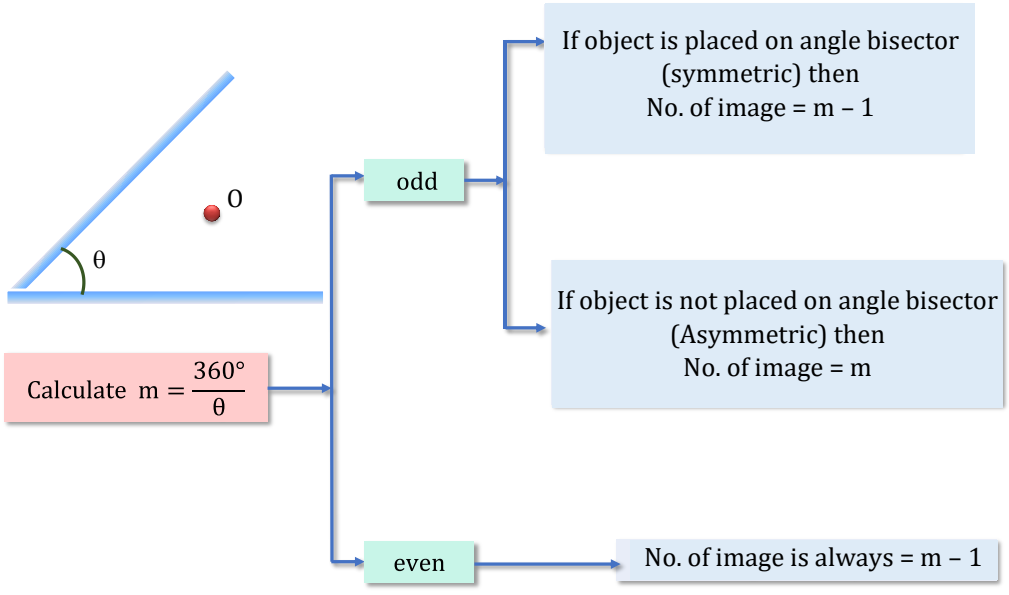
**Solution:**

To solve the question, we use criss-cross method instead of ray-diagram.




Distance between them = 7 + 5 + 12 = 24 cm

**Number of images when mirrors are inclined to each other**


O
θ
Calculate m = 360°/θ
odd
even
If object is placed on angle bisector (symmetric) then No. of image = m – 1
If object is not placed on angle bisector (Asymmetric) then No. of image = m
No. of image is always = m – 1

➢ Previous method is not useful if "m" is in fraction.

**Illustration 17:**

Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror $M_1$. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

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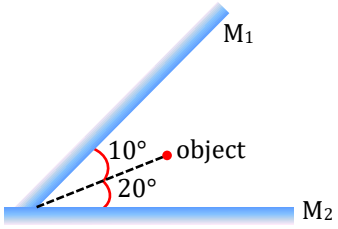
**Solution:**

Number of images

(i) Using direct formula: $\frac{360°}{30°}$ = 12 (even number)

$\therefore$ number of images = 12 – 1 = 11

(ii) By counting. See the following table




| Image formed by Mirror $M_1$ (angles are measured from the mirror $M_1$) | Image formed by Mirror $M_2$ (angles are measured from the mirror $M_2$) |
|---|---|
| 10° | 20° |
| +30° | |
| 50° | 40° |
| +30° | |
| 70° | 80° |
| +30° | |
| 110° | 100° |
| +30° | |
| 130° | 140° |
| +30° | |
| 170° | 160° |
| +30° | |
| Stop because next angle will be more than 180° | Stop because next angle will be more than 180° |

To check whether the final images made by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide.

Here last angles made by the mirrors + the angle between the mirrors = 160° + 170°+30° = 360°. Therefore, in this case the last images coincide. Therefore, the number of images = number of images formed by mirror $M_1$+ number of images formed by mirror $M_2$-1 (as the last images coincide) = 6 + 6 – 1 = 11.

**Note:**

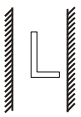
1. Criss-cross method is applicable for any angle between both the mirror.
2. If two mirrors are placed parallel and in front of each other then infinite images are formed of an object placed in between them.
3. When two plane mirrors are inclined to each other, then all the images including object always lie on a circle.

## BEGINNER'S BOX-1

**1.** A boy is 1.8 m tall and can see his image in a plane mirror fixed on a wall. His eyes are 1.6 m from the floor level. The minimum length of the mirror to see his full image is :-

(1) 0.9 m (2) 0.85 m (3) 0.8 m (4) can't be determined

**2.** An object shaped as 'L' is placed between two parallel plane mirrors as shown. In the first seven closest images, how many laterally inverted images are formed ?



(1) 1 (2) 2 (3) 3 (4) 4

**3.** A plane mirror is placed horizontally on level ground at a distance of 60 m from the foot of a tower. Light rays from the top of tower falling just on the edge of the mirror suffers a deviation of 90°. The height of the tower is :-

(1) 30 m (2) 60 m (3) 90 m (4) 120 m

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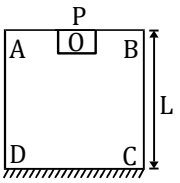
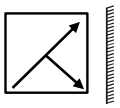
**4.** A plane mirror makes an angle of 30° with the horizontal. If a vertical ray strikes the mirror, find the angle between the mirror and the reflected ray ?

**5.** Figure shows the plane (top view) of a cubical room, with the wall CD as a plane mirror; each side of the room is 3 metres in length. A camera P is placed at the mid point of the wall AB. At what distance should the camera be focused to photograph the image of an object placed at A ?



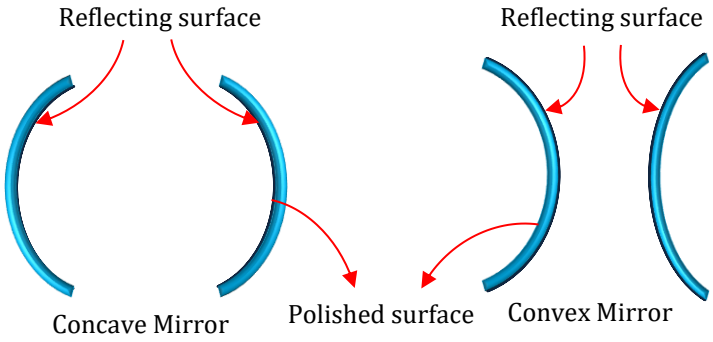

**6.** The correct mirror–image of the figure given is :-



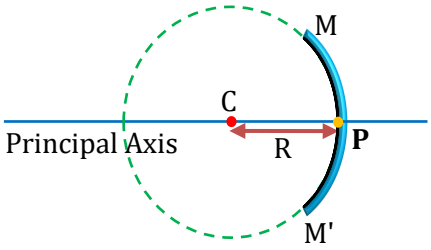
(1)
(2)
(3)
(4)

## Reflection from Spherical Surface (Spherical Mirror)

A spherical mirror is a mirror that has the shape of a piece cut out of a spherical surface. Curved mirror is part of a hollow sphere. If reflection takes place from the inner surface, then the mirror is called concave and if its outer surface acts as reflector it is convex.




### Basic term related to Spherical mirror




C = Centre of curvature

R = Radius of curvature

P = Pole (geometric centre of mirror)

(i) **Pole** is any point on the reflecting surface of the mirror. For convenience we take it as the central point, P of the mirror.

(ii) **Principal–section** is any section of the mirror such as MM' passing through the pole.

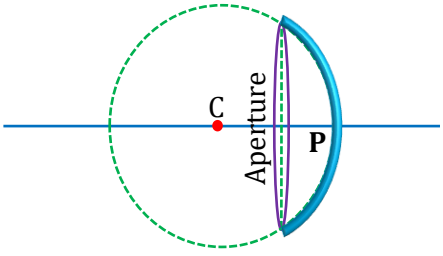
(iii) **Centre of curvature** is the centre C of the sphere of which the mirror is a part.

(iv) **Radius of curvature** is the radius R of the sphere of which the mirror is a part.

(v) **Principal–axis** is the line CP, joining the pole and centre of curvature of the mirror.

### Aperture

It is the effective diameter of the light reflecting area of the mirror.



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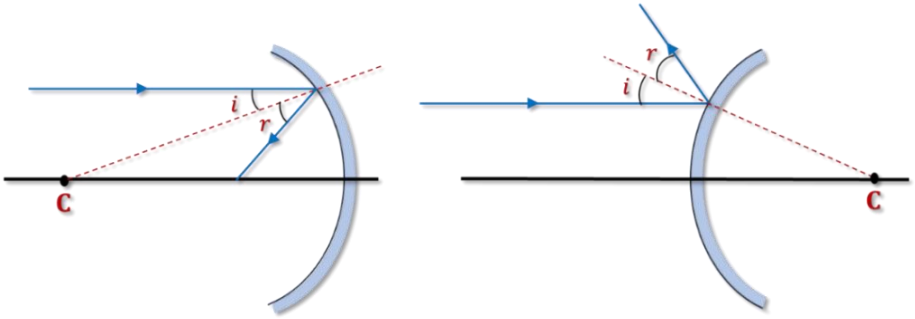
### How to draw the normal in spherical mirror:

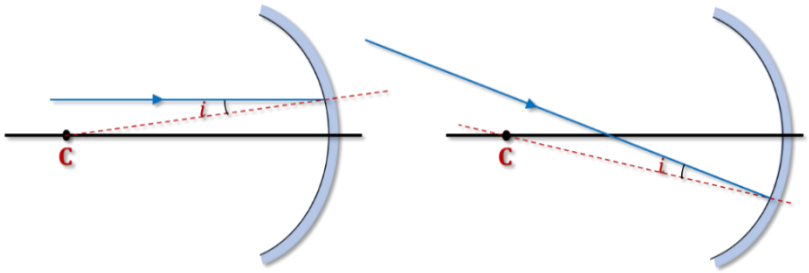
We can draw normal to a spherical mirror at a particular point by joining the particular point with the centre of curvature of the mirror.




Concave mirror (converging mirror)

Convex mirror (diverging)

**Paraxial Rays:** Those rays which make very small angle with normal at point of incidence is called paraxial rays.

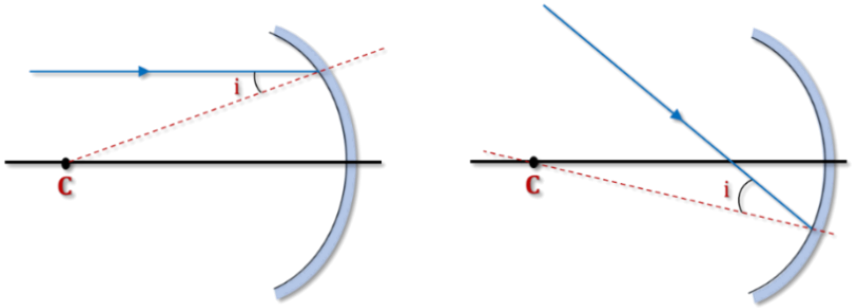



$\angle i$ is small

Paraxial Rays need not to be parallel to principle axis.

**Marginal Rays:** Those rays which make large angle with normal at a point of incidence is called marginal rays.

$\angle i$ is large




### Focal length of a spherical mirror

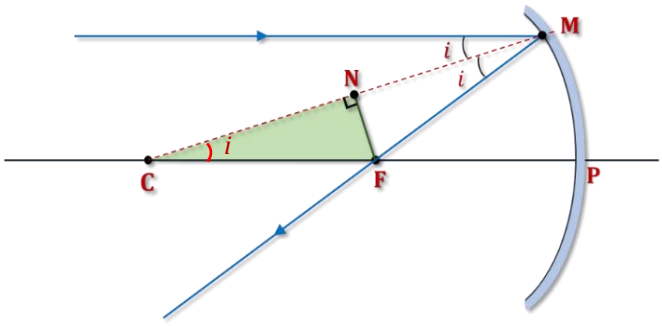
What happens when a parallel beam of light is incident on a concave mirror, or a convex mirror. We assume that the rays are paraxial, i.e., they are incident at points closed the pole P of the mirror and make small angles with the principal axis. The reflected rays converge at a point F on the principal axis of a concave mirror. For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis. The point F is called the principal focus of the mirror.

TG: @Chalnaayaaar

**Derivation:**

$\Delta CNF \sim \Delta NFM$

Then, $CN = NM = \frac{R}{2}$




$$\cos i = \frac{CN}{CF} = \frac{\left(\frac{R}{2}\right)}{CF}$$

$$CF = \frac{R}{2\cos i}$$

$$FP = CP - CF = R - \frac{R}{2\cos i}$$

If i is very small i.e. rays are paraxial, then $\cos i \approx 1$

$$FP = R - \frac{R}{2} \Rightarrow \boxed{FP = \frac{R}{2} = f}$$ (Here, f is called focal length of mirror).

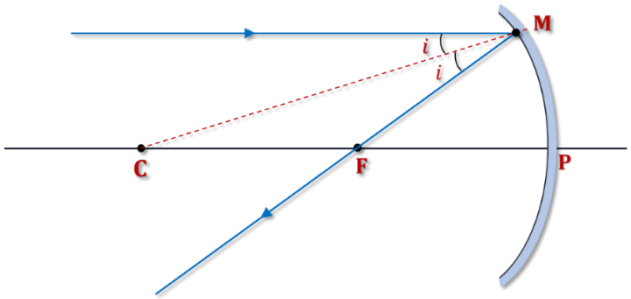
**Conclusion:**

For marginal rays,

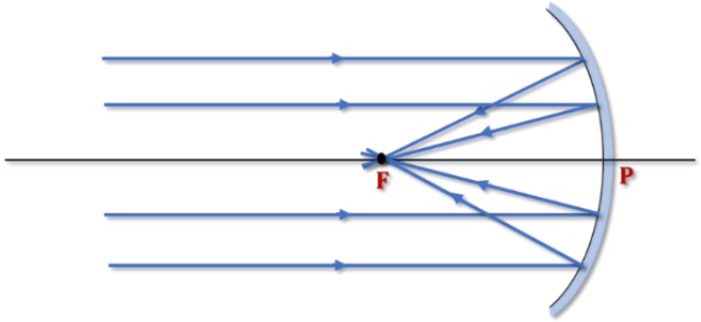
$$\mathbf{FP = R - \frac{R}{2\cos i}}$$

If rays are paraxial then,

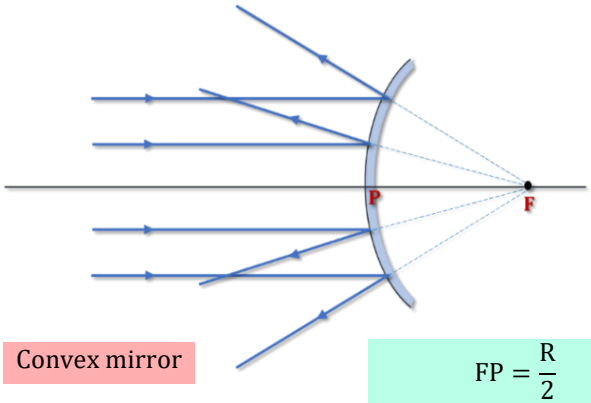
$$\mathbf{FP = f = \frac{R}{2}}$$




➢ When paraxial light rays parallel to principal axis are incident on a mirror then the point where they meet (concave mirror) or appears to meet (convex mirror) after reflection is knows as focus (F).




Concave mirror $FP = \frac{R}{2}$




Convex mirror $FP = \frac{R}{2}$

### Focal Plane

If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the principal axis. This is called the focal plane of the mirror.

If the parallel paraxial beam of light were incident, making some angle with plane passing through focus and perpendicular to the principal axis,

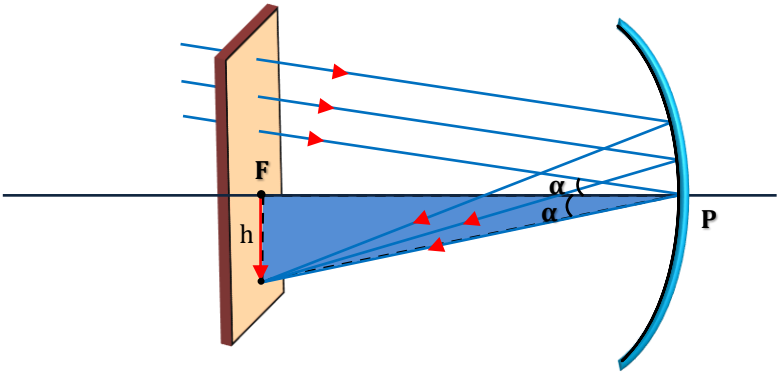
$\tan\alpha = \frac{h}{FP}$

Rays are paraxial so,

$\tan\alpha \approx \alpha$

$\alpha = \frac{h}{f}$

$\boxed{h = f\alpha}$




### Illustration 18:

The sun subtends an angle θ radians at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror?

### Solution:

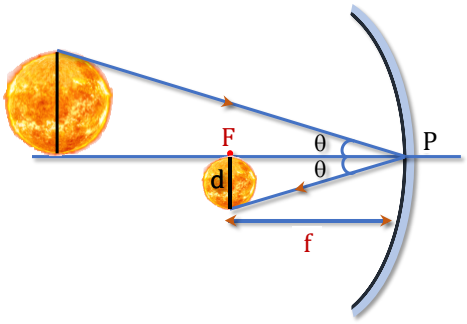
$\tan\theta = \frac{d}{FP}$

Rays are paraxial so,

$\tan\theta \approx \theta$

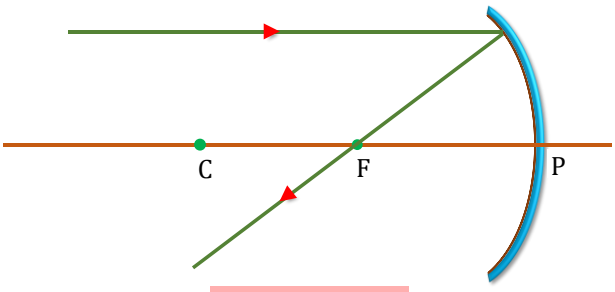
$\theta = \frac{d}{f} \Rightarrow d = \theta f$

$\therefore$ Diameter of image = θf

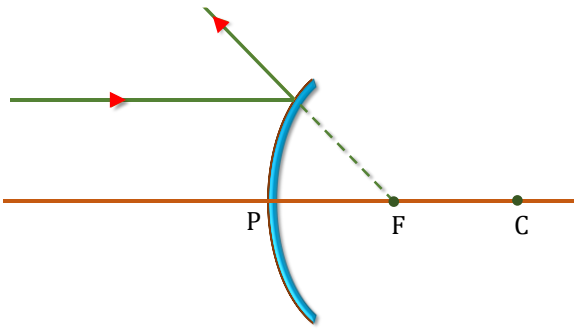



**Rules for image formation:** (for paraxial rays only, based on the laws of reflection)

(1) A light ray parallel to the principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).

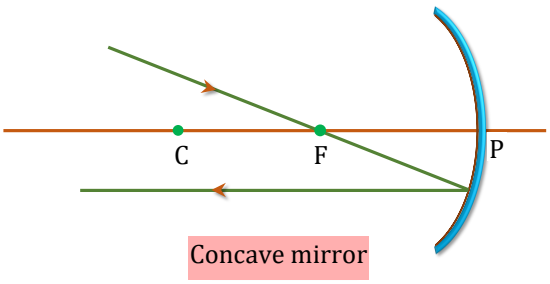



Concave mirror

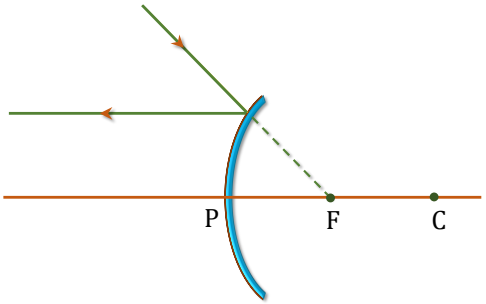



Convex mirror

(2) A light ray passing through or directed towards focus, becomes parallel to the principal axis after reflection from the mirror.

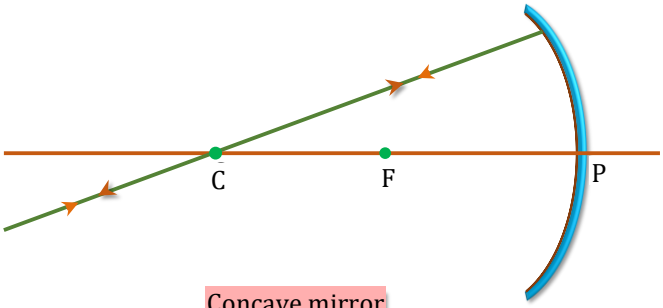



Concave mirror

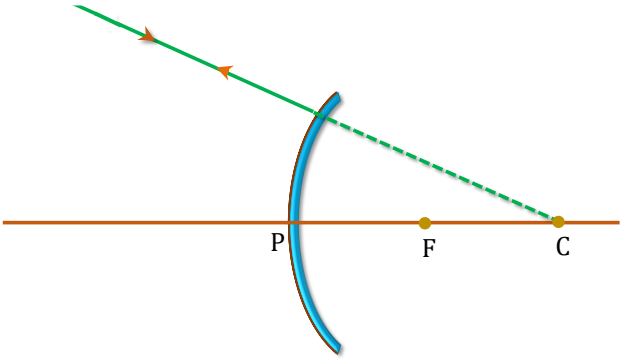



Convex mirror

(3) A ray passing through or directed towards centre of curvature, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$) after reflection from the mirror.

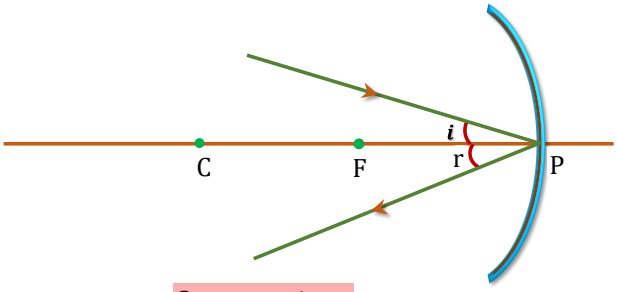



Concave mirror

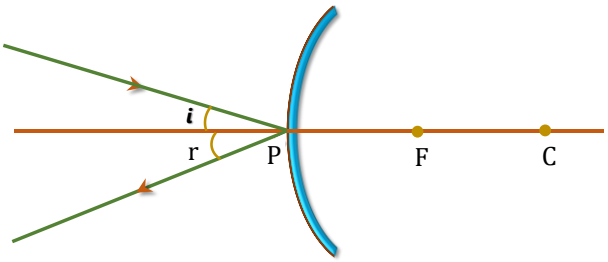



Convex mirror

(4) Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis $\angle i = \angle r$.

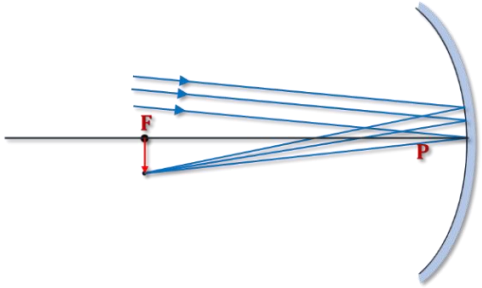



Concave mirror




Convex mirror

## Image Formation by Concave Mirror

| S.N. | Position of Object | Position of Image | Nature | Size | Ray Diagram |
|---|---|---|---|---|---|
| 1. | Place at infinity | Formed at F | Real, Inverted | Highly diminished |  |



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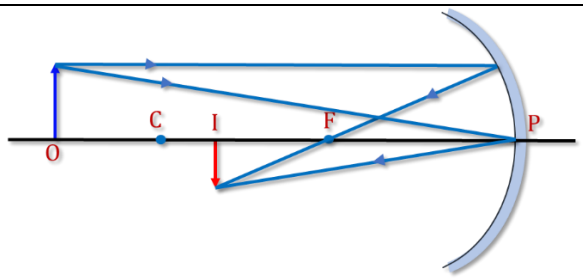
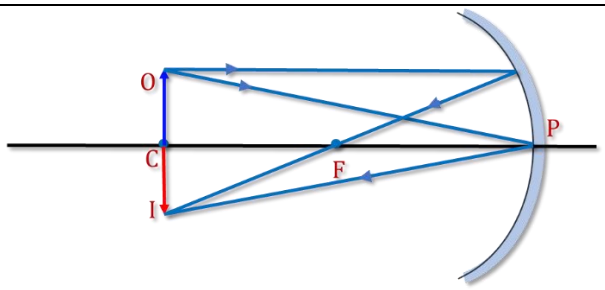
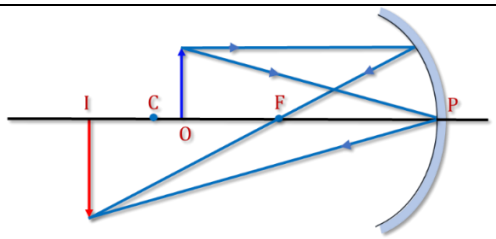
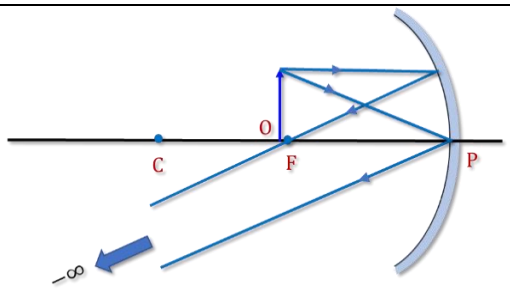
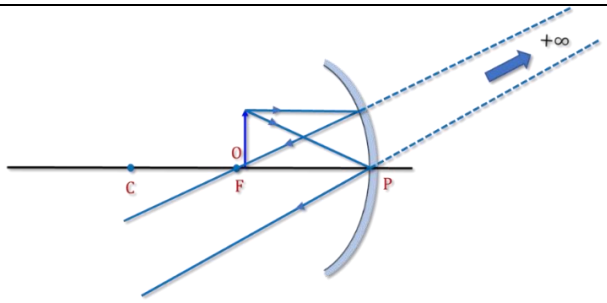
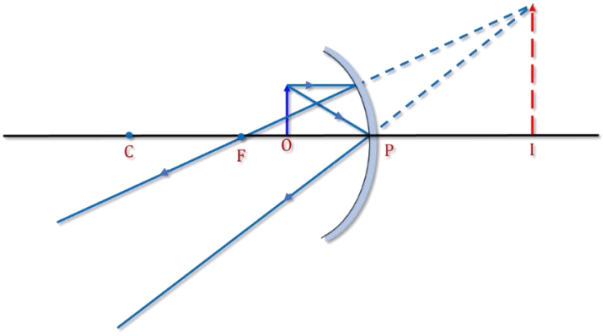
| | | | | | |
|---|---|---|---|---|---|
| **2.** | Placed in between infinity and C | Formed in between C and F | Real, Inverted | Diminished |  |
| **3.** | Placed at C | Formed at C | Real, Inverted | Equal in size |  |
| **4.** | Placed in between F and C | Formed beyond C | Real, Inverted | Enlarged |  |
| **5.** | Placed between F and C, very near to F | Formed at negative infinity | Real, Inverted | Highly enlarged |  |
| **6.** | Placed between F and pole, very near to F | Formed at positive infinity | Virtual, erect | Highly enlarged |  |
| **7.** | Placed between F and P | Formed behind the mirror | Virtual, erect | Enlarged |  |

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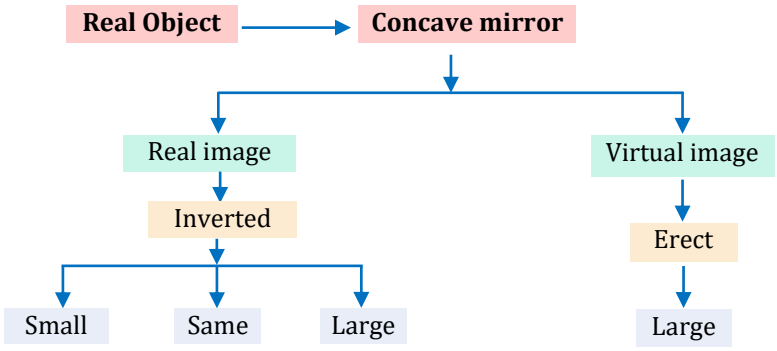
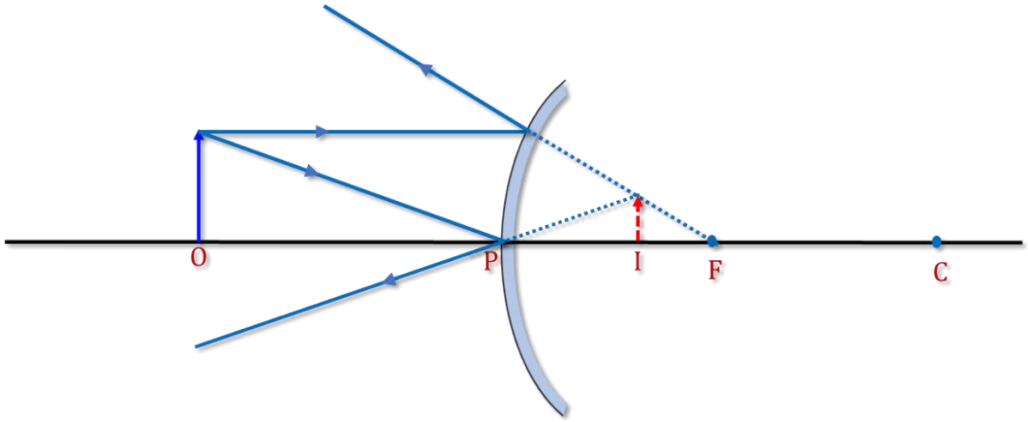
**Things to Keep in mind while solving the problems:**




**Note:** Concave mirror always form real image for virtual object.

### Image formation by convex mirror

Image is always virtual and erect, whatever be the position of the real object.

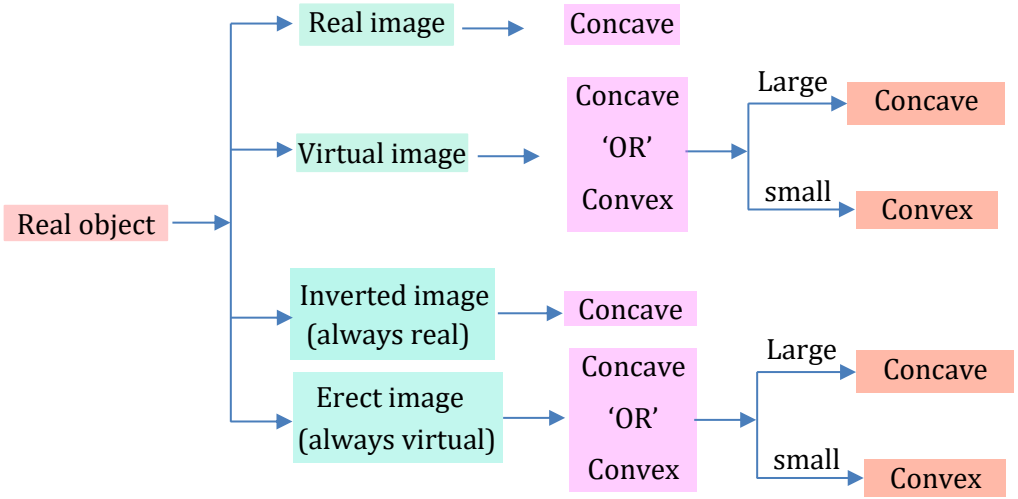



### Keep in mind during solving the problems

Real Object → Convex mirror → Virtual image → Erect → Small

Convex mirror form both real & virtual Image of virtual object, depends on the position of virtual object.

**Things to keep in mind while solving a problem:**



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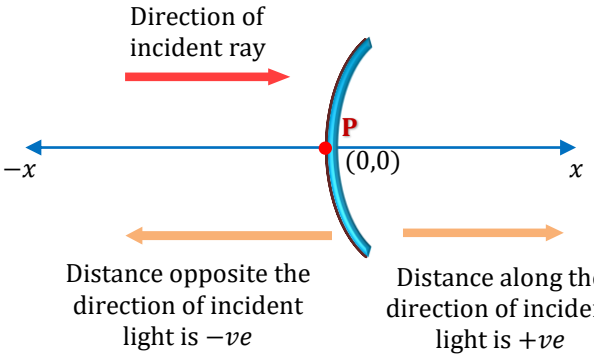
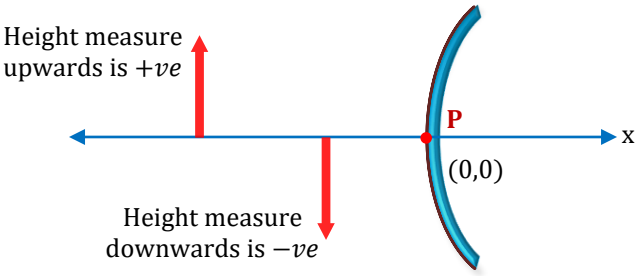
## Sign Convention & Mirror Formula

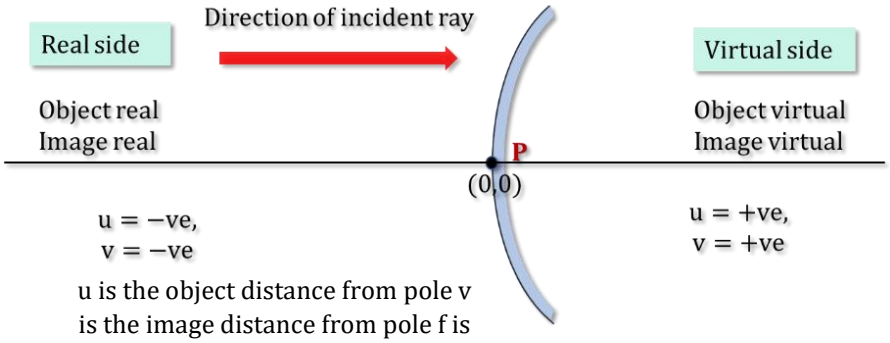
**Sign Convention:** To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses, we must first adopt a sign convention for measuring distances. We shall follow the Cartesian sign convention. According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lenses.




- Along the principal axis, distances are measured from the pole (Pole is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite directions are taken negative.
- The distances above the principal axis are taken positive while below it is negative.
- Whenever and wherever possible, incident light is taken to travel from left to right.




**Things to Keep in mind while solving a problem:**

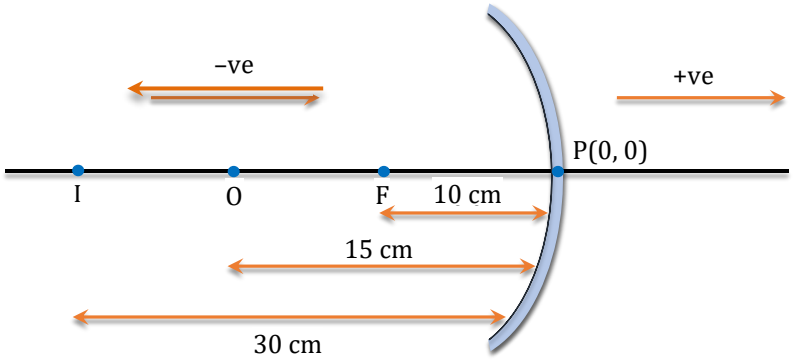



u is the object distance from pole v is the image distance from pole f is the focal length of mirror

### Illustration 19:

A point object is placed at distance 15 cm in front of concave mirror of focal length 10 cm and its image formed at distance 30 cm in front of mirror. Comment on sign of u, v and f :

**Solution:**




**U = negative, v = Negative, f = Negative**

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**Mirror Formula:** Relation between u, v and f in spherical mirror.

An object is placed at a distance u from the pole of a mirror for small angles and its image is formed at a distance v (from the pole).

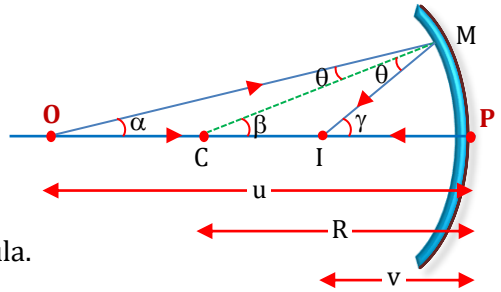
If angle is very small : $\alpha = \frac{MP}{u}, \beta = \frac{MP}{R}, \gamma = \frac{MP}{v}$

From $\Delta CMO$, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

From $\Delta CMI$, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

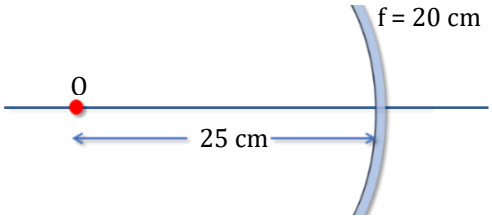
So we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$ This equation is called mirror formula.




### Illustration 20:

Find out position and nature of image?

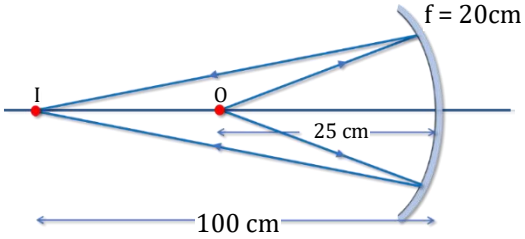



### Solution:

u = – 25 cm, f = – 20 cm, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

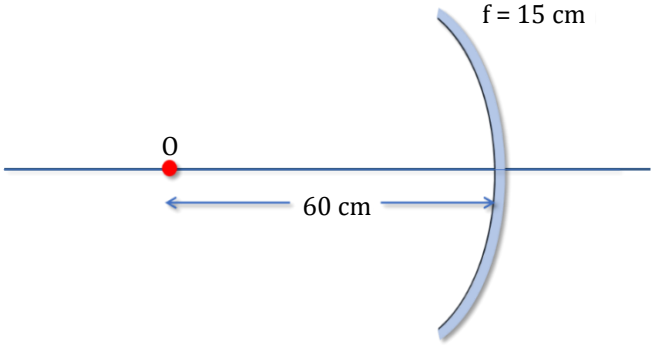
$\frac{1}{v} + \frac{1}{-25} = \frac{1}{-20}$

$\Rightarrow$ v = - 100 cm, real image




### Illustration 21:

Find out position and nature of image?




### Solution:

u = – 60 cm, f = – 15 cm,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

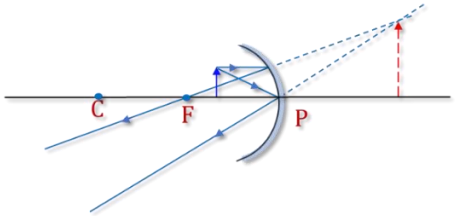
$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-15}$

$\Rightarrow$ v = – 20 cm, real image

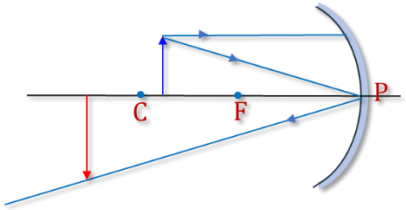
### Transverse or Lateral Magnification

If one dimensional object is placed perpendicular to the principal axis then ratio of image height and object height is called **transverse or lateral magnification**.

$$\boxed{m_t = \frac{h_i}{h_o}}$$




If $m_t$ is positive means erect image is formed




If $m_t$ is negative means inverted image is formed

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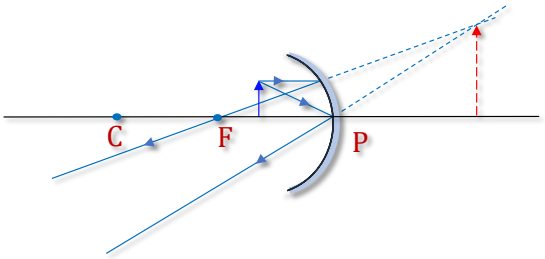
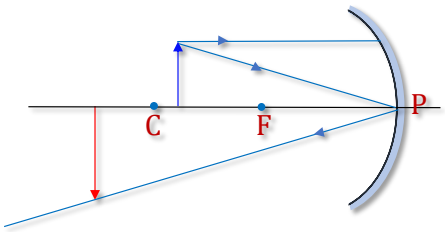
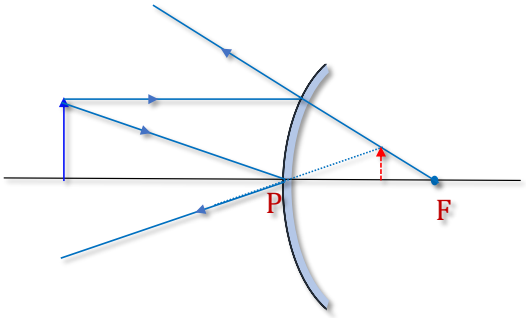
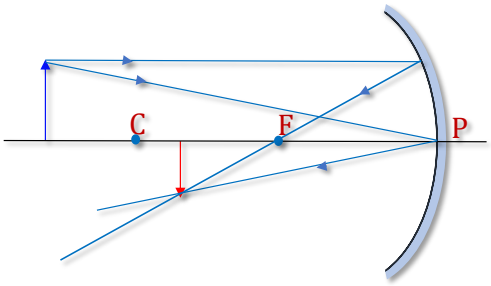


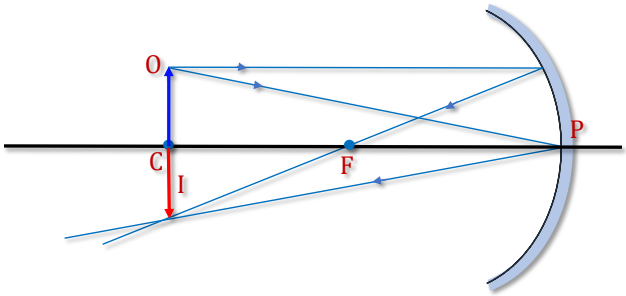




$|m_t| > 1$




$|m_t| < 1$




$|m_t| = 1$

**Derivation of Transverse Magnification:**

$$\frac{h_i}{h_o} = \frac{v}{u}$$

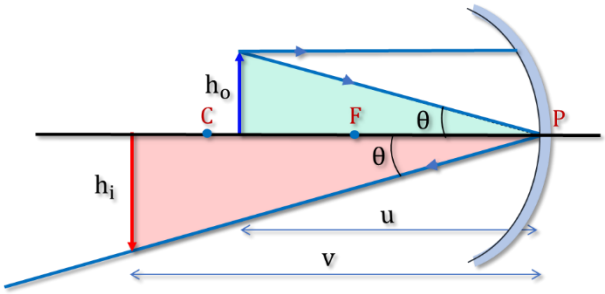
Put the value with sign,

$u \to -u$, $v \to -v$, $h_i \to -h_i$ & $h_o \to +h_o$, then

$$\frac{-h_i}{h_o} = \frac{-v}{-u}$$

$$\frac{h_i}{h_o} = -\frac{v}{u}$$

$$\boxed{m_t = \frac{h_i}{h_o} = -\frac{v}{u}}$$




$$\tan\theta = \frac{h_o}{u}, \quad \tan\theta = \frac{h_i}{v} \Rightarrow \frac{h_i}{h_o} = \frac{v}{u}$$

**Important Results:** $m_t = \frac{h_i}{h_o}$

- $m_t = \frac{h_i}{h_o} = -\frac{v}{u}$
- $m_t = \frac{h_i}{h_o} = \frac{f}{f-u} = \frac{f-v}{f}$

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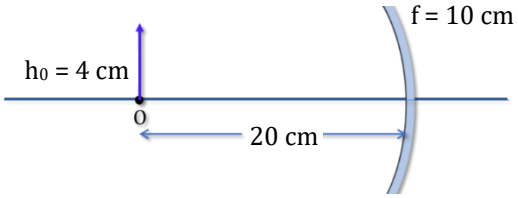
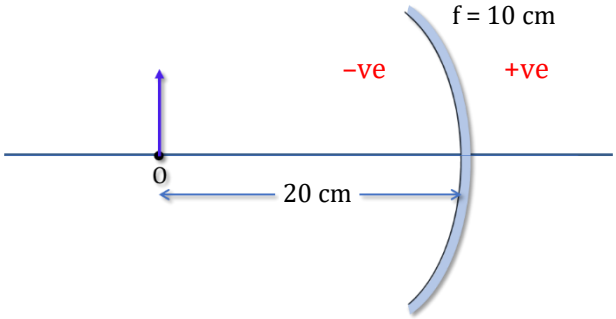
### Illustration 22:

Find out position, nature & size of image of the object as shown in figure.




### Solution:




u = –20 cm ; f = – 10 cm

$m = \frac{f}{f-u} = \frac{-v}{u}$

$m = \frac{-10}{-10-(-20)} = \frac{-v}{-20}$

V = –20 cm

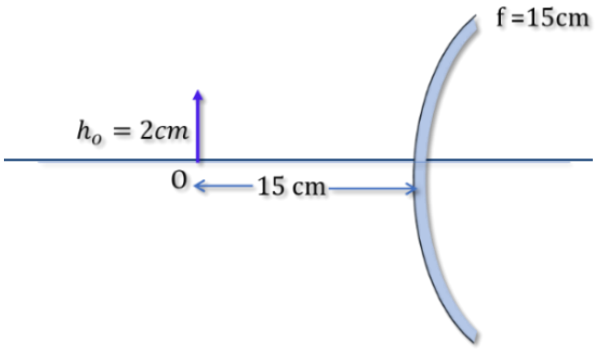
$m = \frac{h_I}{h_O} = \frac{h_I}{4}$

$h_I$ = –4 cm

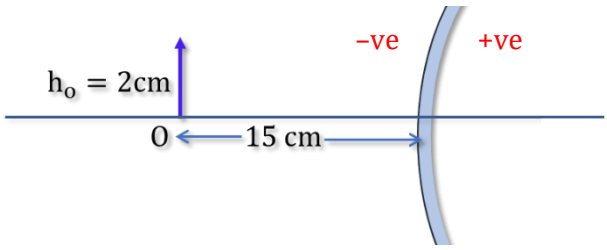
Real inverted and of same size as that of object.

### Illustration 23:

Find out position, nature & size of image of the object as shown in figure.




### Solution:




Given; u = –15 cm ; f = 15 cm

$m = \frac{f}{f-u} \Rightarrow m = \frac{15}{15-(-15)} = \frac{1}{2}$

$m = \frac{-v}{u} \Rightarrow \frac{1}{2} = \frac{-v}{-15}$

v = 7.5 cm

$h_i = m \times h_o$

$h_i = \frac{1}{2} \times 2 = 1$ cm

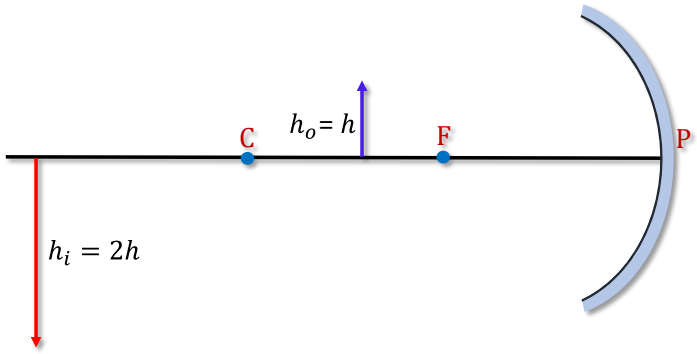
Virtual erect and smaller than object.

### Illustration 24:

Find out position of an object placed in front of concave mirror of focal length 30cm so that 2 times magnified image is formed.

### Solution:

**Case-1**




Given; $m = -2$

$$-2 = \frac{f}{f-u}$$

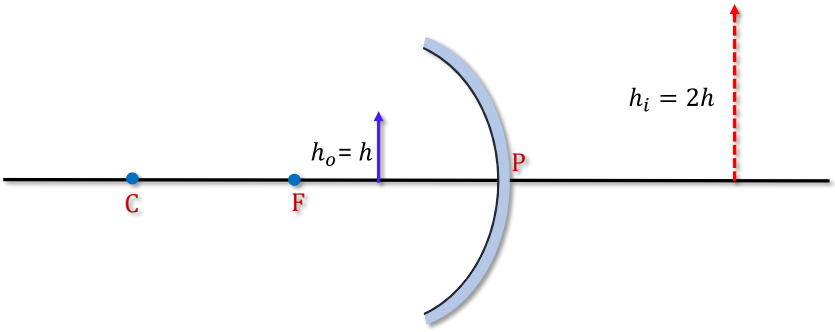
$$-2 = \frac{-30}{-30-u}$$

$$60 + 2u = -30$$

$$2u = -90$$

$$u = -45 \text{ cm}$$

**Case-2**




Given; $m = +2$

$$+2 = \frac{f}{f-u}$$

$$2 = \frac{-30}{-30-u}$$

$$-60 - 2u = -30$$

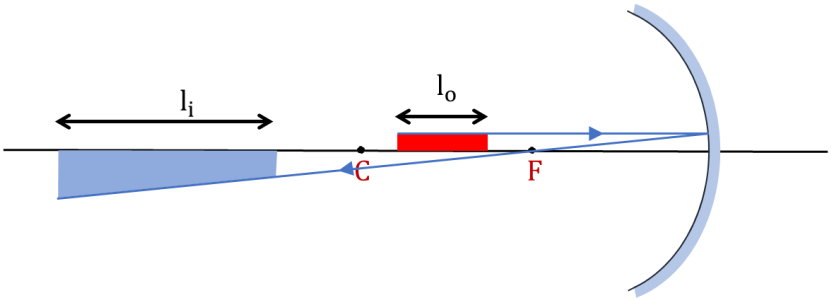
$$2u = -30$$

$$u = -15 \text{ cm}$$

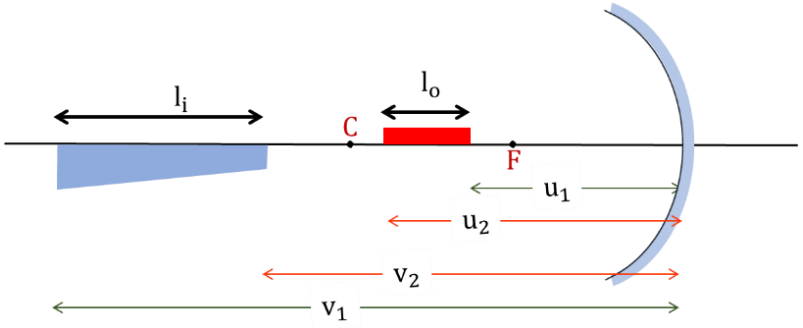
### Longitudinal Magnification

If an object is placed along the principal axis then ratio of length of image and length of object is called longitudinal or axial magnification.

$$m_L = \frac{\text{length of image}}{\text{length of object}} = \frac{l_i}{l_o}$$



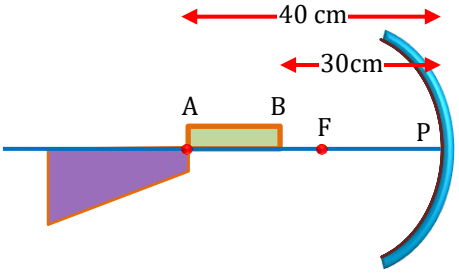

$$m_L = \frac{\text{length of image}}{\text{length of object}} = \left|\frac{v_1 - v_2}{u_2 - u_1}\right|$$



### Illustration 25:

A thin rod of length 10 cm is placed along the principal axis of a concave mirror of focal length 20 cm such that its image which is real and elongated, just touches one end of the rod. Find out the length of image?

### Solution:




For point A; u = –40 cm, f = –20 cm

$$v_A = \frac{u \times f}{u - f} = \frac{-40 \times -20}{-40 + 20} = -40\text{cm}$$

For point B; u = –30 cm, f = –20 cm

$$v_A = \frac{uf}{u - f} = \frac{-30 \times -20}{-30 + 20} = -60\text{cm}$$

Length of image = 60 – 40 = 20 cm.

**Longitudinal magnification for small objects ($L_o << f$):**

object length = du; Image length = dv

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating w.r.t . u

$$-\frac{1}{v^2}\frac{dv}{du} - \frac{1}{u^2} = 0 \Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2}$$

If we use only magnitude then $\frac{dv}{du} = \frac{v^2}{u^2}$

$$\boxed{M_L = \frac{l_i}{l_o} = \frac{v^2}{u^2} = m_t^2}$$

### Illustration 26:

A point object is placed at a distance of 15 cm from a concave mirror of radius of curvature 20 cm. If the object is made to oscillate along the principal axis with amplitude 2 mm. Find out the amplitude of its image.

### Solution:

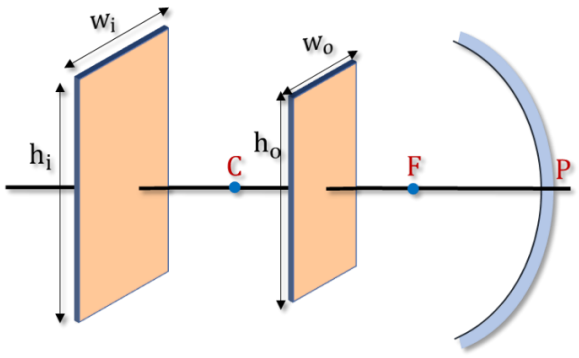
Amplitude of object |du| = 2 mm

$$m = \frac{f}{f - u} = \frac{-10}{-10 - (-15)} = -2$$

amplitude of image |dv| = $m^2$ |du| = $(-2)^2 \times 2 = 8$ mm

### Superficial Magnification

If two-dimensional object is placed with its plane perpendicular to the principal axis then its magnification is known as superficial magnification.




$$m_s = \frac{A_i}{A_O}$$

$A_o = w_o h_o$ and $A_i = w_i h_i$

$$m_t = \frac{h_i}{h_o} = \frac{w_i}{w_o}$$

$$\frac{A_i}{A_o} = \frac{w_i h_i}{w_o h_o}$$

$$m_t = \frac{h_i}{h_o} = \frac{w_i}{w_o}$$

$$\boxed{\frac{A_i}{A_o} = m_t^2 = m_s}$$

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**Illustration 27:**

A square of side 4 mm is placed at a distance of 30 cm from a concave mirror of focal length 10 cm. The centre of the square is at the axis of the mirror and the plane is normal to the axis. Find the area enclosed by the image?

**Solution:**

$$m_t = \frac{f}{f-u} = \frac{-10}{-10-(-30)} = -\frac{1}{2}$$

$$\frac{A_i}{A_o} = m_t^2 \Rightarrow \frac{A_i}{(4mm)^2} = \left(-\frac{1}{2}\right)^2 \Rightarrow A_i = 4mm^2$$

**Newton's Formula**

$x_1$ : Object distance from focus

$x_2$ : Image distance from focus

The relation between $x_1$, $x_2$, f is,

$$\boxed{f = \sqrt{x_1 x_2}}$$

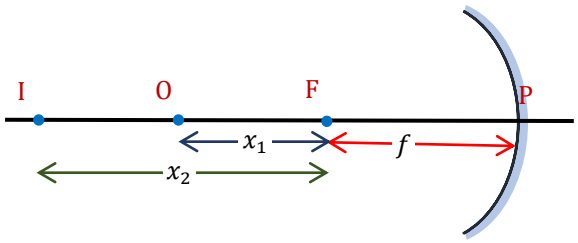



**Illustration 28:**

If an object is placed at a distance 30 cm from focus of a concave mirror of focal length 60 cm, find distance between image and focus of mirror.

**Solution:**

Given, $x_1 = 30$ cm; $f = 60$ cm

So, $f^2 = x_1 x_2 \Rightarrow (60)^2 = 30\, x_2 \Rightarrow x_2 = 120$ cm

**Velocity of image in spherical mirror**

**Case (1):** When object is moving along the principal axis of spherical mirror.

From equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Differentiate with respect to time,

$$-\frac{1}{u^2}\frac{du}{dt} + \frac{-1}{v^2}\frac{dv}{dt} = 0$$

$$\frac{1}{v^2}\frac{dv}{dt} = -\frac{1}{u^2}\frac{du}{dt}$$

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

$\boxed{\vec{V}_{IM} = -m_t^2\, \vec{V}_{OM}}$ All velocities are instantaneous.

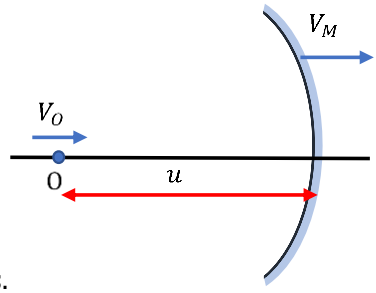



**Illustration 29:**

Find out velocity of image.

**Solution:**

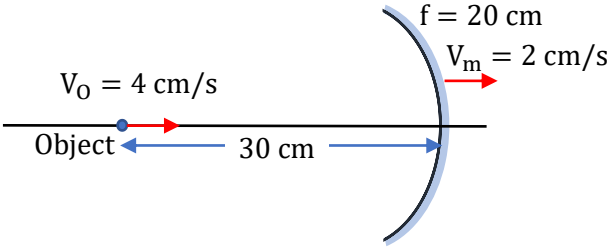
$$m_t = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

$$\vec{V}_{IM} = -m_t^2\, \vec{V}_{OM}$$

$$\vec{V}_I - \vec{V}_M = -m_t^2\left(\vec{V}_O - \vec{V}_M\right)$$

$$\vec{V}_I - 2 = -4(4-2)$$

$\vec{V}_I = -6$ cm/s ; So, image moves with speed 6 cm/s away from the mirror.

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**Case (2):** When object is moving perpendicular to the principal axis of spherical mirror.

From equation $\frac{h_i}{h_o} = -\frac{v}{u}$

Differentiate with respect to time,

$\frac{dh_i}{dt} = -\frac{v}{u}\frac{dh_o}{dt}$

$\boxed{\vec{V}_{IM} = m_t \vec{V}_{OM}}$

All velocities are instantaneous.

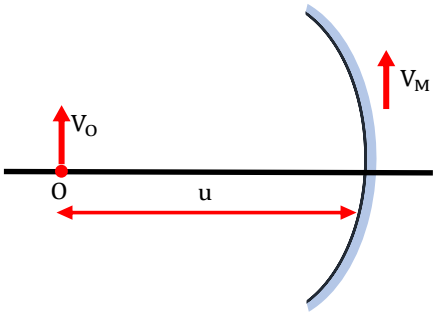



**Illustration 30:**

Find out velocity of image.

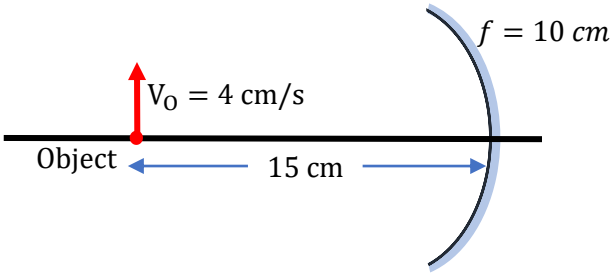
**Solution:**

$m_t = \frac{f}{f-u} = \frac{-10}{-10+15} = -2$

$\vec{V}_{IM} = m_t \vec{V}_{OM}$

$\vec{V}_I - \vec{V}_M = m_t\left(\vec{V}_O - \vec{V}_M\right)$

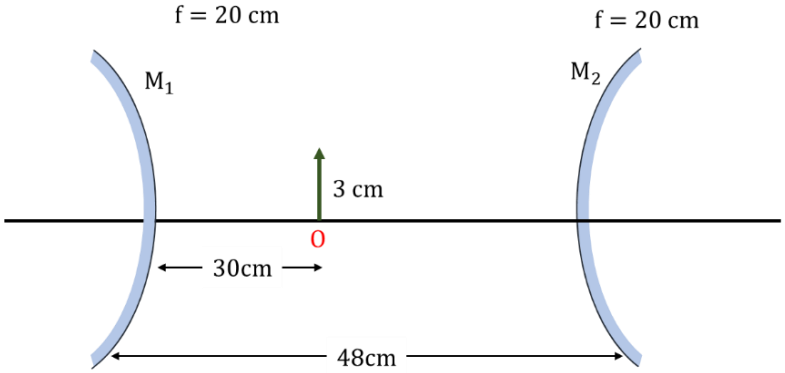
$\vec{V}_I - 0 = -2(4) \Rightarrow \vec{V}_I = -8\ cm/s$




## Combination of Mirrors

**Illustration 31:**

Find out height of image formed after second reflection (consider first reflection on mirror $M_1$)




**Solution:**

For mirror $M_1$ (First reflection)

u = –30 cm, f = 20 cm

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-30}=\frac{1}{20}$

v = 12 cm

$\frac{h_{I_1}}{h_o} = -\frac{v}{u} \Rightarrow \frac{h_{I_1}}{3} = -\frac{12}{-30}$

$h_{I_1} = 1.2$

$\frac{h_{I_2}}{h_{I_1}} = -\frac{15}{-60} \Rightarrow \frac{h_{I_2}}{1.2} = -\frac{15}{-60}$

$h_{I_2} = 0.3$ cm

for mirror $M_2$

u = –60 cm, f = 20 cm

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}+\frac{1}{-60}=\frac{1}{20}$

v = 15 cm

Total magnification $= \frac{\text{Height of final image}}{\text{Height of object}} = \frac{0.3}{3} = \frac{1}{10}$

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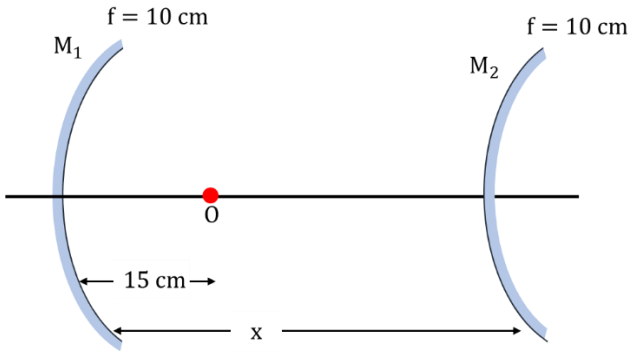
**Note:** In case of successive reflection from mirrors, the overall lateral magnification is given by $m_1 \times m_2 \times m_3$ ....., where $m_1$, $m_2$ etc. are lateral magnifications produced by individual mirrors.

**Illustration 32:**

Find out the value of x such that light ray retraces its path after reflection. (Consider first reflection on mirror $M_1$)




**Solution:**

For mirror $M_1$, u = –15 cm, f = –10 cm

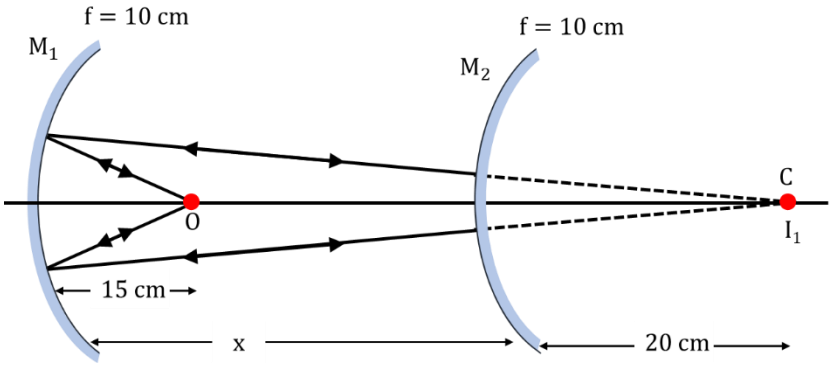
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{-15}=\frac{1}{10}$$

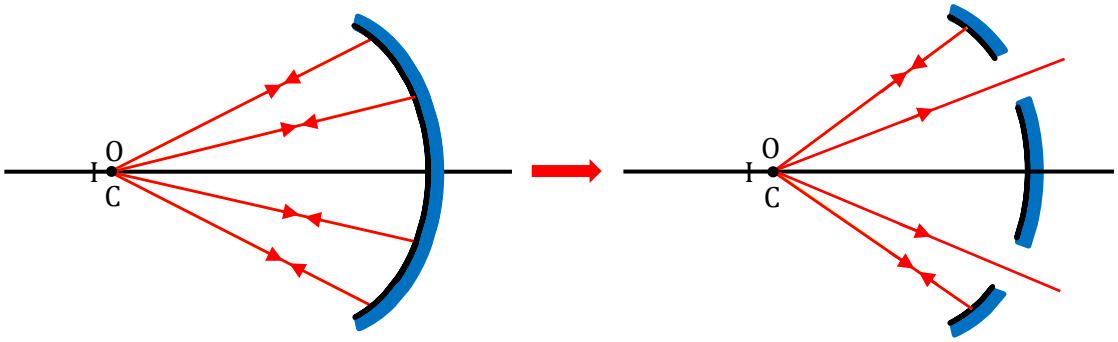
v = – 30 cm

For the light ray to retrace it's path, image of mirror $M_1$ should formed at centre of curvature of mirror $M_2$

$\therefore$ x + 20 = 30 $\Rightarrow$ x = 10 cm



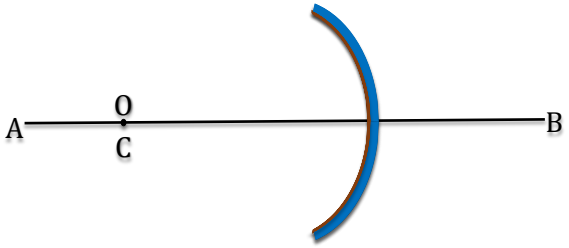

**Cutting of mirror**




All the parts of mirror have same hollow sphere, so its centre of curvature is same therefore no of images found is 1. Intensity of image decreases.

**Illustration 33:**

An object is placed at the Centre of curvature of concave mirror of focal length f as shown in figure. Now concave mirror is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1mm from the previous principal axis AB. Find the distance between the images formed by the two parts?

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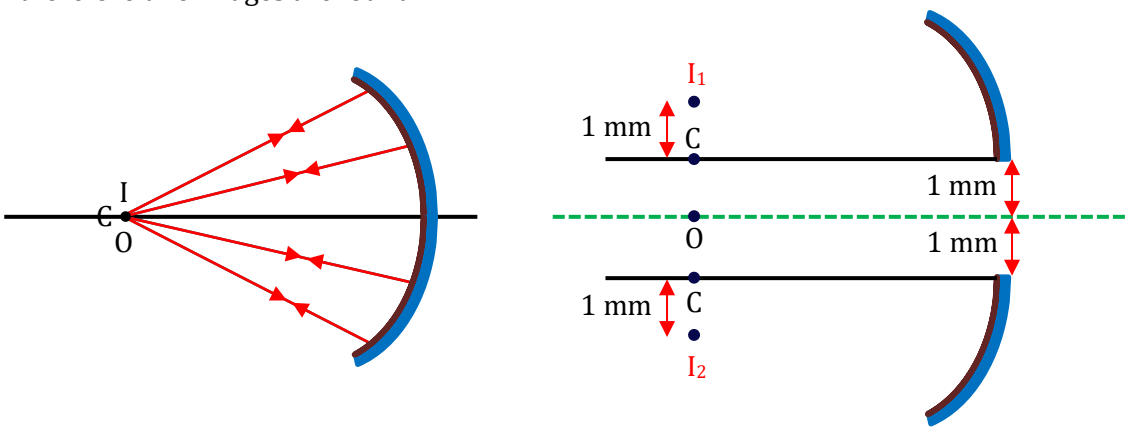
**Solution:**

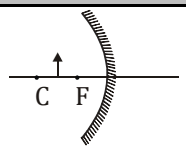
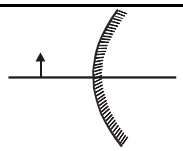
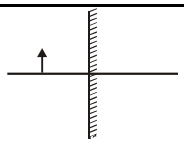
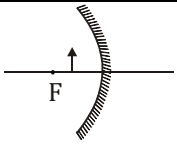
A concave mirror is cut and each part is shifted by 1 mm. Then centre of curvature of each part shift by 1 mm and each part behaves as two independent concave mirror with its centre of curvature at the new position therefore two images are found.




Distance between images = 4 mm

## BEGINNER'S BOX-2

**1.** Column-I contains a list of mirrors along with the position of object. Match this with Column-II describing the nature of the image.

| Column I | Column II |
|---|---|
| (A)   | (P) real, inverted, enlarged |
| (B)  | (Q) virtual, erect, enlarged |
| (C)  | (R) virtual, erect, diminished |
| (D)   | (S) virtual, erect |

**2.** A man has a shaving mirror of focal length 0.2 m. How far should the mirror be held from his face in order to give an image of two fold magnification ?

**3.** A convex mirror has a focal length f. A real object is placed at a distance of $\frac{f}{2}$ from the pole. Find out the position, magnification and nature of the image.

**4.** A motor car is fitted with a rear view mirror of focal length 20 cm. A second motor car 2 m broad and 21.6 m high is 6 m away from first car. Find the position of second car as seen in the mirror of the first car.

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**5.** A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. Find the distance of the object from the mirror.

**6.** The focal length of a concave mirror is 30 cm. Where should an object be placed so that its image is three times magnified, real and inverted ?

**7.** A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image ? Describe the nature and size the image. If the candle is moved closer to the mirror, how should the screen have to be moved ?

## Refractive index ($\mu$)

The refractive index of the material is an indicator of its optical density.

**Optical density** $\uparrow \Rightarrow \mu\uparrow \Rightarrow v\downarrow$

Absolute Refractive Index or simply refractive index of a medium is defined as the ratio of the speed of light in a vacuum (c) to the speed of light in that medium (v).

$$\boxed{\mu = \frac{c}{v}} \qquad \mu \geq 1$$

**Example:** for vacuum $\mu = 1$ ; for air $\mu_a \approx 1$

for glass $\mu_g = 3/2$ ; for water $\mu_w = 4/3$

- $\mu$ is a scalar quantity and has no unit and dimension.

## Relative refractive index

When light passes from one medium to another medium, then refractive index of second medium with respect to the first medium is known as relative refractive index.

$${}_1\mu_2 = \mu_{21} = \frac{\mu_2}{\mu_1}$$ $\mu_{21}$ = Refractive index of medium-2 with respect to medium-1.

$${}_2\mu_1 = \mu_{12} = \frac{\mu_1}{\mu_2}$$ $\mu_{12}$ = Refractive index of medium-1 with respect to medium-2.

$$\Rightarrow \quad \boxed{\mu_{21} = \frac{1}{\mu_{12}}} \quad \Rightarrow \quad \boxed{\mu_{21} \times \mu_{12} = 1}$$

Refractive index of water with respect to glass, ${}_g\mu_w = \mu_{wg} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$

Refractive index of glass with respect to water, ${}_w\mu_g = \mu_{gw} = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

From the above example we can say that $\mu_{relative}$ may be greater than or less than 1.

**Note:** (1) Both denser and rarer are relative term

- The medium with a **higher refractive index** in which **speed of light** is comparatively **less** is known as **optically denser** medium.
  **For example:** Water is denser medium as compared to air.
- The medium with **lower refractive index** in which the **speed of light** is comparatively **more** is known as **optically rarer** medium.

**For example:** Water is rarer medium as compared to glass.

(2) Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media) i.e. both optical density and mass density are different term.

**For example:**

turpentine

mass density ($\rho_T$) < mass density ($\rho_W$)

Optical Density (O.D.)$_T$ > Optical Density (O.D.)$_W$

$V_T$ < $V_W$

Water

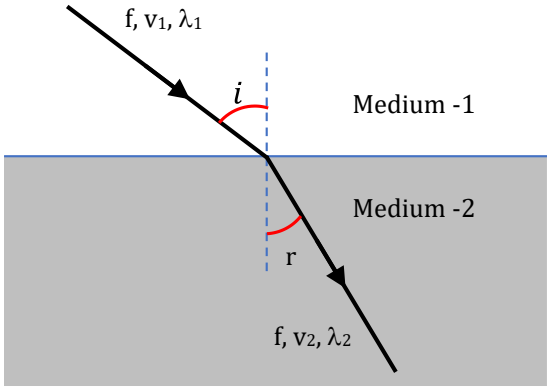
- If $\varepsilon_0$ and $\mu_0$ are electric permittivity and magnetic permeability of free space respectively while $\varepsilon$ and $\mu$ are those of a given medium, then according to electromagnetic theory,

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}} \text{ and } v_m=\frac{1}{\sqrt{\varepsilon\mu}} \Rightarrow n_m=\frac{c}{v_m}=\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}=\sqrt{\varepsilon_r\mu_r}$$

- As in vacuum or free space, speed of light of all wavelengths is maximum and equal to c, so for all wavelengths the refractive index of free space is minimum and is $\mu=\frac{c}{v_m}=\frac{c}{c}=1$.

## Refraction of Light

Refraction is the phenomenon in which ray of light bends when it passes obliquely from one medium (such as air) into another medium (such as glass) in which its velocity is different.




i is the angle of incidence.
r is the angle of refraction.

When light rays goes from one medium to another medium then speed (v) and wavelength ($\lambda$) changes but frequency (f) remains same.

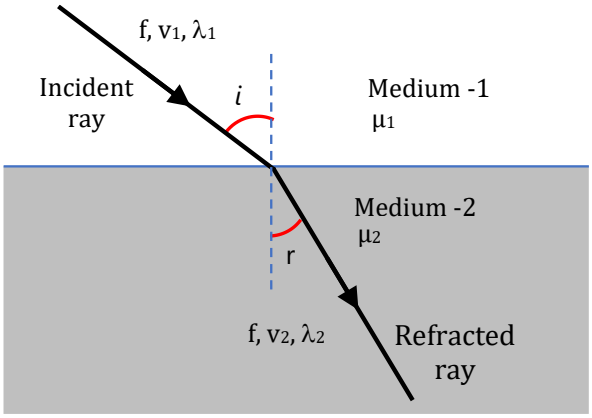
### Laws of Refraction

(i) Incident ray, refracted ray and normal always lie in the same plane.

(ii) The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant, for the light of a given colour and for the given pair of media.

$$\boxed{\frac{\sin i}{\sin r}=\text{constant}} \text{ (Snell's Law)}$$

$$\frac{\sin i}{\sin r}=\mu_{21} \Rightarrow \frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1} \Rightarrow \boxed{\mu_1 \sin i=\mu_2 \sin r}$$



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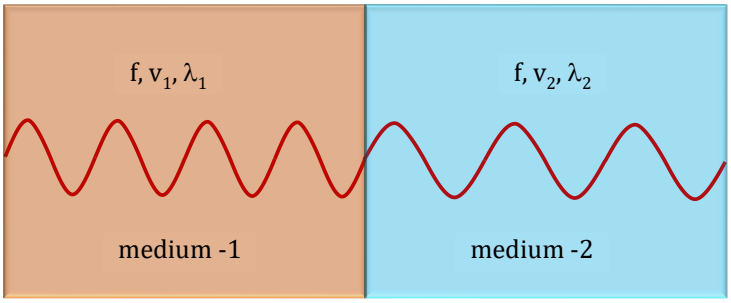



From, $f = \frac{v}{\lambda}$

in medium-1 $f = \frac{v_1}{\lambda_1}$

in medium-2 $f = \frac{v_2}{\lambda_2}$

Now $\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$ ...(i)

$\left(\text{From, } v = \frac{c}{\mu}\right)$

$\frac{c/\mu_1}{\lambda_1} = \frac{c/\mu_2}{\lambda_2} \Rightarrow \boxed{\lambda_1\mu_1 = \lambda_2\mu_2}$

$\frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1}$ ...(ii)

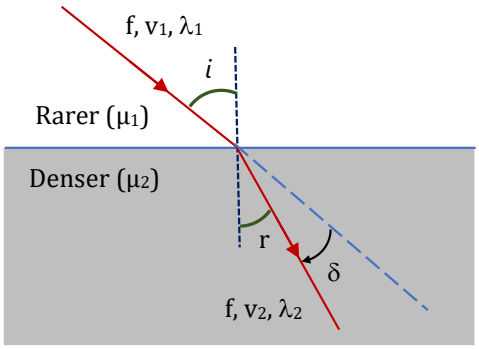
From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \boxed{\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}}$

## Bending of light ray

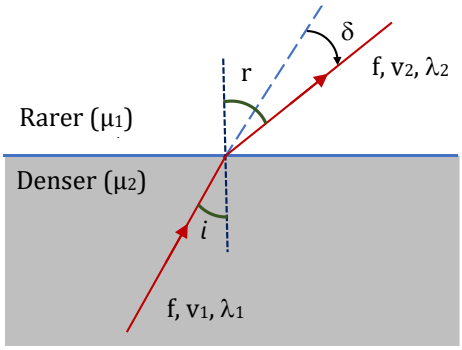
**Case -1:** When light rays goes from rarer medium to denser medium ($\mu_2 > \mu_1$).

- Light rays bend towards the normal after refraction.
- $\angle i > \angle r$
- $v_1 > v_2$
- $\lambda_1 > \lambda_2$
- $\delta = i - r$




**Case -2:** When light rays goes from denser medium to rarer medium ($\mu_2 > \mu_1$)

- Light rays bend away from the normal after refraction.
- $\angle i < \angle r$
- $v_1 < v_2$
- $\lambda_1 < \lambda_2$
- $\delta = r - i$




**Note:** If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

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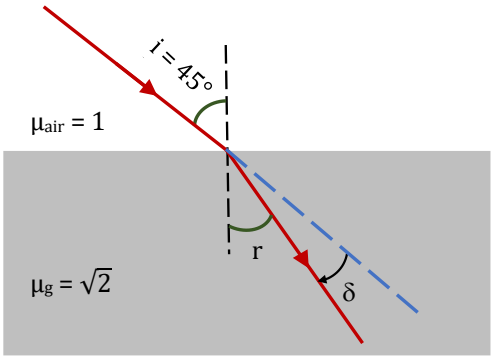
**Illustration 34:**

A light ray incident on a glass surface of refractive index $\sqrt{2}$ from air at an angle of incidence 45°.

Find out: (i) Angle of refraction (ii) Angle of deviation.

**Solution:**




(i) From, Snell's law,

$\mu_1 \sin i = \mu_2 \sin r$

$1 \sin 45° = \sqrt{2} \sin r \Rightarrow \frac{1}{2} = \sin r$

$r = 30°$

(ii) Angle of deviation $\delta = i - r = 45° - 30°$

$\delta = 15°$ (CW)

**Illustration 35:**

A light ray is moving from air to denser medium of refractive index μ, it is observed that the angle of refraction is half of the angle of incidence then find out angle of incidence.

**Solution:**

By Snells law, $\mu_1 \sin i = \mu_2 \sin r$

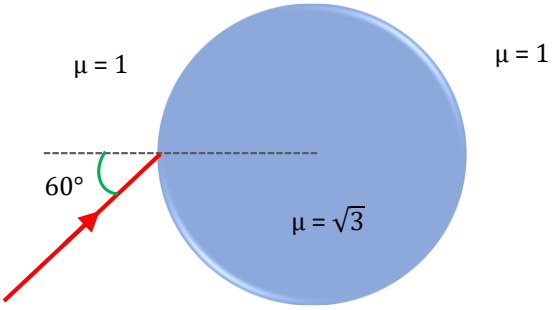
$1 \sin i = \mu \sin r \quad \left(\text{Given, } r = \frac{i}{2}\right)$

$\sin i = \mu \sin \frac{i}{2} \quad \left(\because \sin 2\theta = 2\sin\theta\cos\theta\right)$

$2\sin\frac{i}{2}\cos\frac{i}{2} = \mu \sin\frac{i}{2} \Rightarrow \cos\frac{i}{2} = \frac{\mu}{2} \Rightarrow \frac{i}{2} = \cos^{-1}\left(\frac{\mu}{2}\right) \Rightarrow i = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

**Illustration 36:**

Find out total angle of deviation after two refraction through glass sphere of refractive index $\sqrt{3}$.




**Solution:**

1st Refraction: by snell's law

$1 \times \sin 60° = \sqrt{3} \sin r$

$r = 30°$

$\delta_1 = i - r = 60° - 30° = 30°$ (Clockwise)

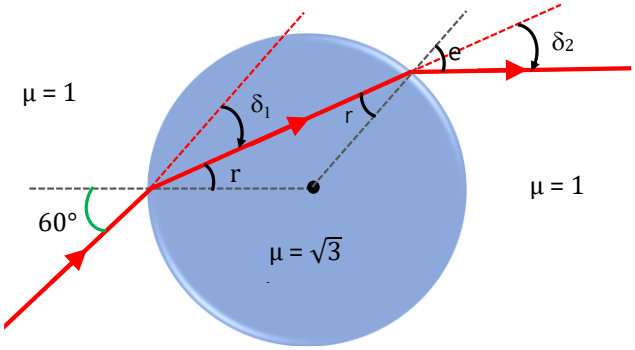
2nd Refraction: by snell's law

$\sqrt{3} \sin r = 1 \times \sin e$

$\sqrt{3} \sin 30° = \sin e \Rightarrow e = 60°$

$\delta_2 = e - r = 60° - 30° = 30°$ (Clockwise)

Total deviation, $\delta = \delta_1 + \delta_2 = 30° + 30° = 60°$ (Clockwise)

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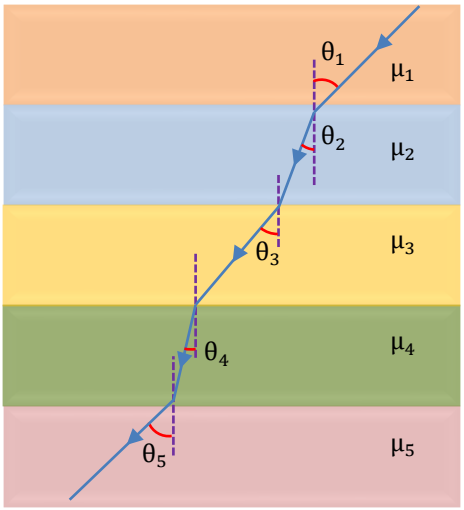
**Note:**

**(a)** **Refraction through Multiple Parallel Slabs:** Consider a stack of five parallel slabs placed vertically one after the other. Since the light ray falls on the topmost slab at an angle of $\theta_1$ with the normal, the angle of refraction at the topmost slab is $\theta_2$ as shown in the figure.

Suppose that the refractive indices of the five slabs are $\mu_1$, $\mu_2$, $\mu_3$, $\mu_4$ and $\mu_5$, respectively, from top to bottom. Since the angle of refraction at the topmost slab is $\theta_2$, the angle of incidence at the second slab is the same. Now, if the angle of refraction at the second slab is $\theta_3$, the angle of incidence for the third slab is the same and this process continues as shown in the figure.




By applying Snell's law of refraction at the interface between the top two slabs, we get,

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$ ...(i)

Now, by applying Snell's law of refraction at the interface between the second and third slab from the top we get,

$\mu_2 \sin\theta_2 = \mu_3 \sin\theta_3$ ...(ii)

By equating equations (i) and (ii), we get,

$\mu_1 \sin\theta_1 = \mu_3 \sin\theta_3$

By going through the same procedure, we get the following result:

$\mu_1 \sin\theta_1 = \mu_4 \sin\theta_4$

In place of five parallel slabs, if there are m numbers of parallel slabs, then we can consider the following equation; $\boxed{\mu_1 \sin\theta_1 = \mu_m \sin\theta_m}$

Where $\mu_m$ is the refractive index of the $m^{th}$ slab, and $\theta_m$ is the angle of incidence at the $m^{th}$ slab.

**(b)** When first and last medium are same then the incident ray and emergent ray are parallel to each other.

## Refraction through single plane surface (Apparent Depth)

If a point object in denser medium is observed from rare medium and boundary is plane.

From Snell's Law

$\mu_1 \sin i = \mu_2 \sin r$

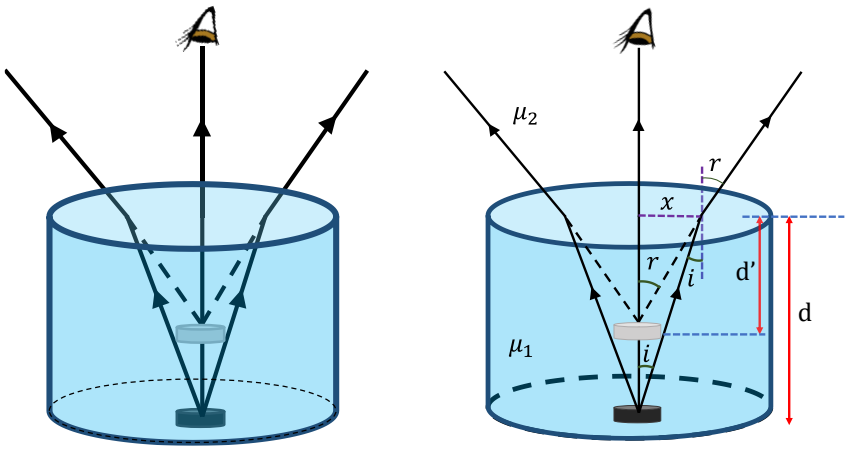
$\sin r \approx \tan r = \frac{x}{d'}$

$\sin i \approx \tan i = \frac{x}{d}$

$\mu_1 \frac{x}{d} = \mu_2 \frac{x}{d'}$

$\boxed{\frac{d'}{d} = \frac{\mu_2}{\mu_1}}$

$\boxed{\frac{\text{Apparent distance}}{\text{Actual distance}} = \frac{d'}{d} = \frac{\mu_2}{\mu_1}}$

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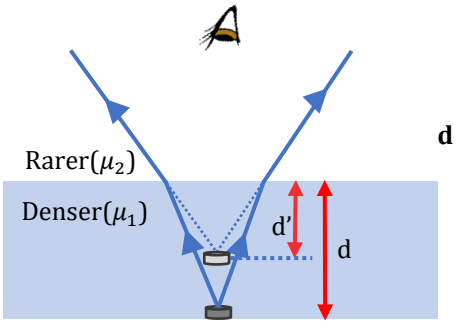
## Important points

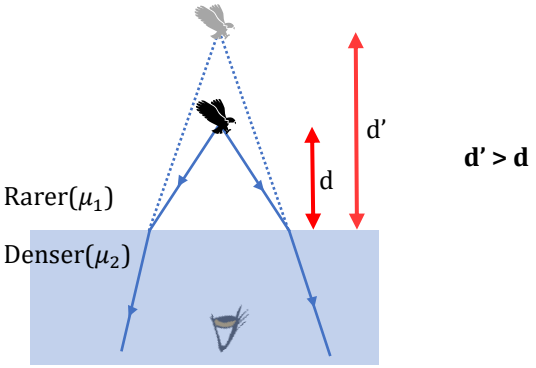
1. Rays should be paraxial. (Rays which are very near to the line joining the object and observer).
2. Both d and d' are measured from boundary.
3. $\mu_2$ is the refractive index of that medium in which light rays enter after refraction and $\mu_1$ is the refractive index of that medium in which light rays moves before refraction.

## Different situations of observer

**Case 1:** If observer is in rarer medium and object is in denser medium,

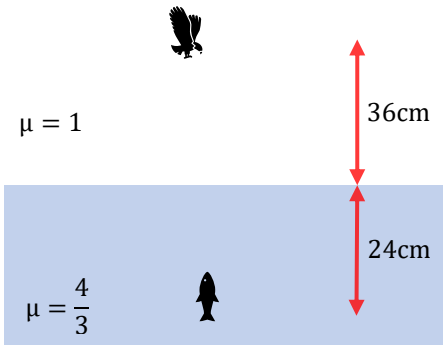



**Case 2:** If observer is in denser medium and object is in rarer medium




## Illustration 37:

Find out apparent distance between bird and fish as seen by bird.




## Solution:

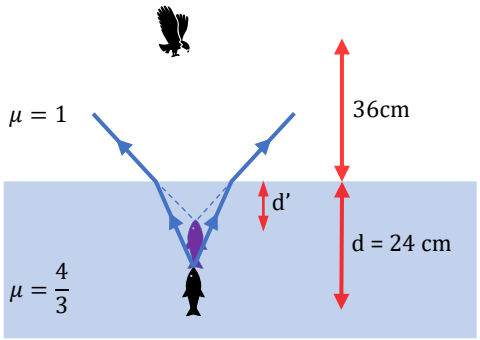
In this case object is fish and observer is bird.

$$\frac{d'}{d}=\frac{\mu_2}{\mu_1}\Rightarrow\frac{d'}{24}=\frac{1}{4/3}$$

d' = 18 cm (from the surface)

Apparent distance between bird and fish as seen by bird

= d' + 36 = 18 + 36 = 54 cm

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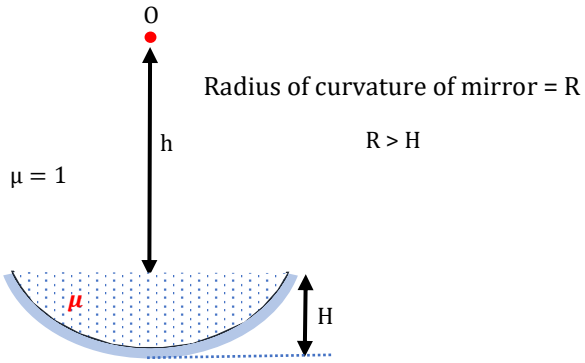
### Illustration 38:

Find out the value of h such that light ray retrace its path after reflection.

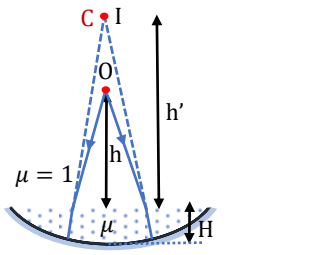



### Solution:

1st Refraction,

$$\frac{d'}{d}=\frac{\mu_2}{\mu_1}\Rightarrow\frac{h'}{h}=\frac{\mu}{1}\Rightarrow h'=\mu h$$

Light ray retrace its path after reflection when image formed by refraction is at the centre of curvature of the mirror.




So, $H+h'=R$

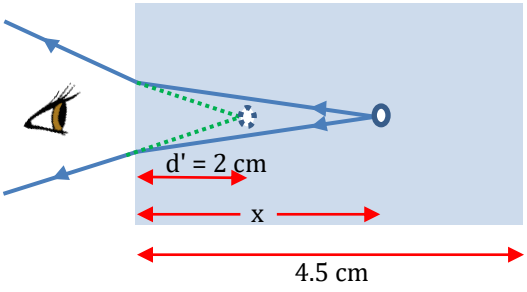
$$H+\mu h=R$$

$$h=\frac{R-H}{\mu}$$

### Illustration 39:

An air bubble inside a cubical block of glass of side 4.5 cm seems to be at 2 cm from one face and 1 cm from the other face opposite to the first when viewed normally. What is the real distance of bubble from the first face.

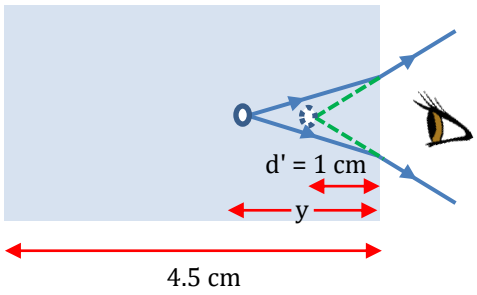
### Solution:




$$\frac{d'}{d}=\frac{\mu_2}{\mu_1}$$

$$\frac{2}{x}=\frac{1}{\mu}$$

$$x = 2\mu$$




$$\frac{d'}{d}=\frac{\mu_2}{\mu_1}$$

$$\frac{1}{y}=\frac{1}{\mu}$$

$$y = \mu$$

$x + y = 4.5$

$2\mu + \mu = 4.5 \Rightarrow \mu = 1.5$

Now, $x = 2\mu = 2 \times 1.5 = 3$ cm

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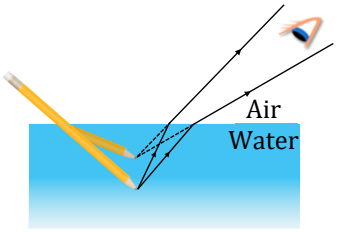
### Examples of refraction

**Bending of an object:** When pencil in a denser medium is seen from a rarer medium it appears to be bent.




**Twinkling of stars:** Due to fluctuations in the refractive index of different layers of atmosphere, the refraction becomes irregular so, that the light sometimes reaches the eye and sometimes it does not. This gives the effect of twinkling of stars.

### Velocity of image in plane refraction

$$\frac{d'}{d}=\frac{\mu_2}{\mu_1}\Rightarrow d'=\frac{\mu_2}{\mu_1}d$$

Differentiate with respect to time,

$$\frac{d(d')}{dt}=\frac{\mu_2}{\mu_1}\frac{d(d)}{dt}$$

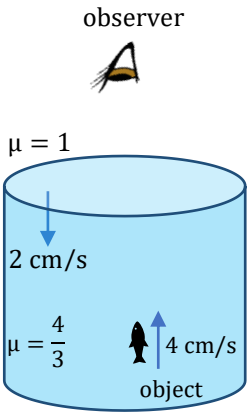
$$\boxed{\vec{V}_{IS}=\frac{\mu_2}{\mu_1}\vec{V}_{OS}}$$

$V_{IS}$ = velocity of image with respect to surface (boundary)

$V_{OS}$ = velocity of object with respect to surface (boundary)

### Illustration 40:

Find out speed of fish as seen by observer if water level decreases with speed 2 cm/s and fish is moving upward with speed 4 cm/s.




**Solution:**

$$\vec{V}_{IS}=\frac{\mu_2}{\mu_1}\vec{V}_{OS}$$

$$\vec{V}_I-\vec{V}_S=\frac{\mu_2}{\mu_1}\left(\vec{V}_0-\vec{V}_S\right)$$

$$\vec{V}_I-(-2)=\frac{1}{4/3}(4-(-2))$$

$$\vec{V}_I+2=\frac{3}{4}\times 6=\frac{9}{2}$$

$$\vec{V}_I=\frac{9}{2}-2=\frac{5}{2}\text{cm/s}$$

$\vec{V}_I$ is the velocity of image with respect to ground but here velocity of observer is zero so, velocity of fish as seen by observer = $\frac{5}{2}$ cm/s (upward).

### Refraction through glass slab (Normal Shift)

**Normal Shift:** When an object is placed in front of a transparent slab then it appears for an observer shift (S) as shown in figure. Distance between object and image is S or we can say object appears to shift at a distance S. This distance is called Normal shift.

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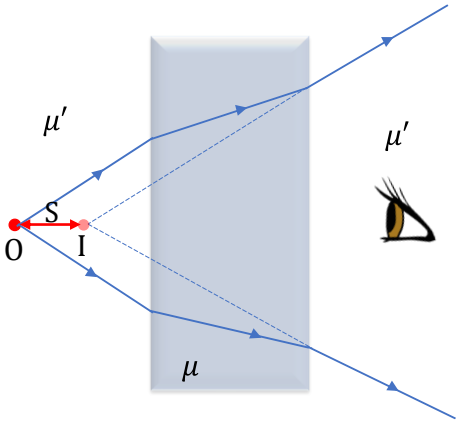
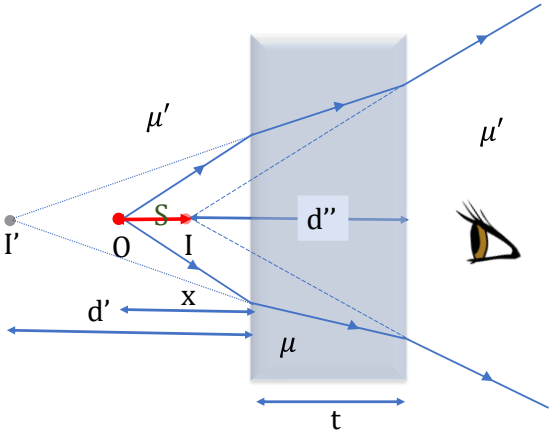



**Calculation of normal shift,**

From $\frac{d'}{d}=\frac{\mu}{\mu'}$

**1st Refraction**

$\frac{d'}{x}=\frac{\mu}{\mu'} \Rightarrow d'=\frac{\mu}{\mu'}x$ (distance I' from 1st surface )

**2nd Refraction:** For second refraction image of first refraction will behave as an object.

$$\frac{d''}{d'+t}=\frac{\mu'}{\mu} \Rightarrow d''=\frac{\mu'}{\mu}(d'+t)$$

Distance between object and its image, $S=d'+t-d''=d'+t-\frac{\mu'}{\mu}(d'+t)$

$$S=t\left(1-\frac{\mu'}{\mu}\right)$$

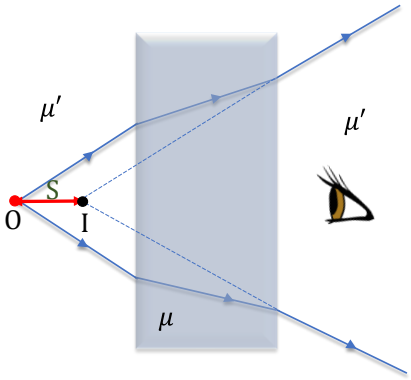
## Conclusion

Shift $\boxed{S=t\left[1-\frac{\mu_{surrounding}}{\mu_{slab}}\right]}$

$$S=t\left[1-\frac{\mu'}{\mu}\right]$$

If surrounding medium is air then, $\mu'=1$

$$\boxed{S=t\left[1-\frac{1}{\mu}\right]}$$




## Important points

1. Rays should be paraxial.
2. Both object and observer are in same medium.
3. Shift is independent of distance between object and slab.
4. If shift comes to be positive then object appears to shift towards the direction of incident ray and vice versa.

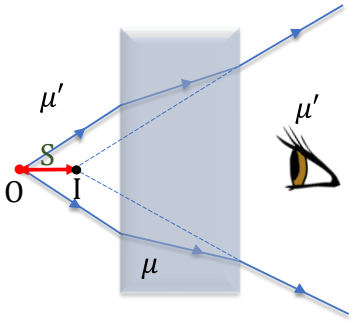



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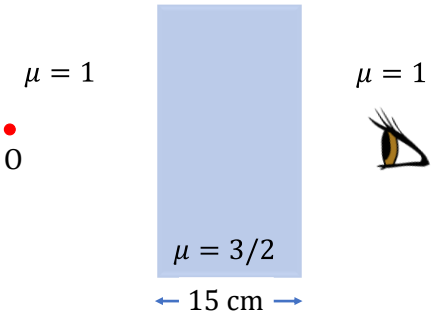
**Illustration 41:**

Find out normal shift.




**Solution:**

$$\text{Normal shift } S = t\left(1-\frac{1}{\mu}\right) = 15\left(1-\frac{1}{3/2}\right) = 5\text{cm} \text{ (Positive)}$$

Image formed towards the slab or object appears to shift 5 cm towards the slab for an observer.

**Illustration 42:**

A microscope is focused on a coin lying at the bottom of a beaker. The microscope is now raised up by 1 cm. To what depth should the water be poured into the beaker so that coin is again in the focus. ($\mu_w$= 4/3)

**Solution:**

$$S = t\left(1-\frac{1}{\mu}\right)$$

$$1 = t\left(1-\frac{1}{4/3}\right)$$

$$t = 4 \text{ cm}$$

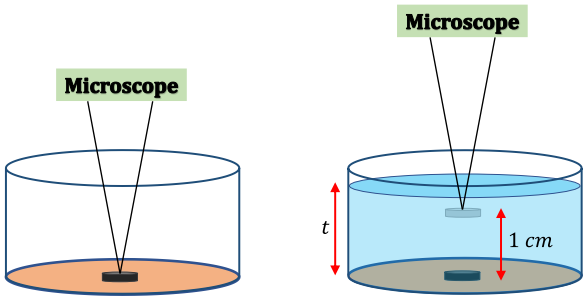
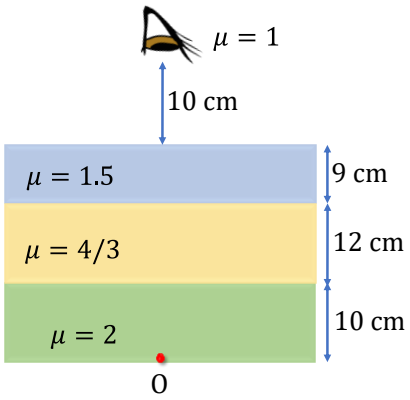



**Illustration 43:**

Find out apparent distance between observer and object.




**Solution:**

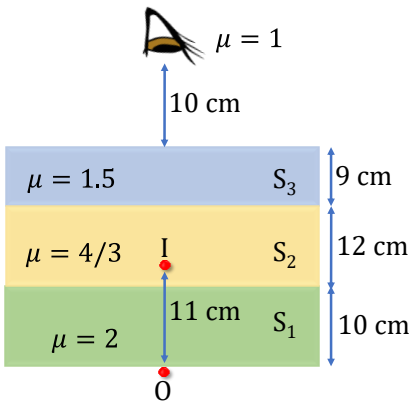
$$S_1 = t\left(1-\frac{1}{\mu}\right) = 10\left(1-\frac{1}{2}\right) = 5 \text{ cm}$$

$$S_2 = 12\left(1-\frac{1}{4/3}\right) = 3 \text{ cm}$$

$$S_3 = 9\left(1-\frac{1}{15}\right) = 3 \text{ cm}$$

Total shift S = 5 + 3 + 3 = 11 cm

Apparent distance between object & observer = 41 – 11 = 30 cm



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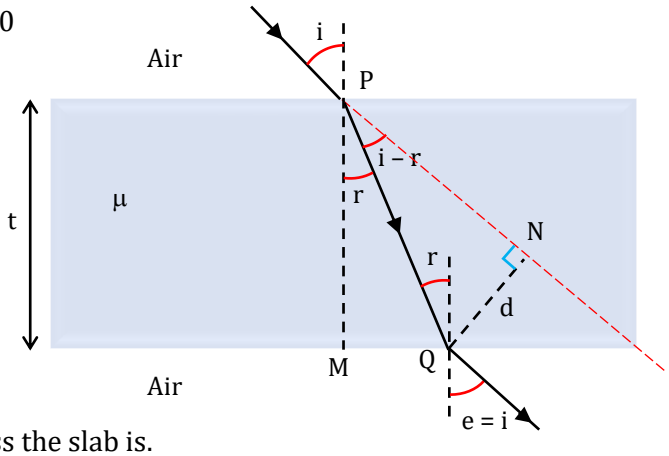
## Refraction Through Glass Slab (Lateral Shift)

**Lateral Shift:** For a rectangular slab, refraction takes place at two interfaces. It is easily seen from the dig. that the emergent ray is parallel to the incident ray-there is no deviation, but it does suffer lateral displacement/shift with respect to the incident ray. The perpendicular distance between incident and emergent ray is known as lateral shift (d).

Deviation (δ) between incident and emergent ray = 0




**Calculation of Lateral Shift**

In $\Delta PQM,\ \cos r = \frac{PM}{PQ} \Rightarrow PQ = \frac{t}{\cos r}$

In $\Delta PQN,\ \sin(i-r) = \frac{QN}{PQ}$

$QN = PQ\sin(i-r)$

$$\boxed{d = \frac{t}{\cos r}\sin(i-r)}$$

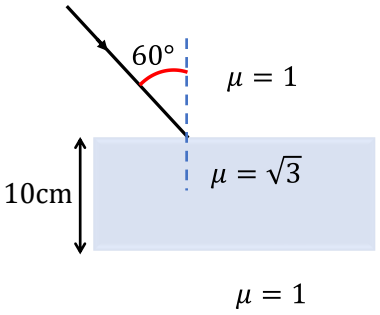
**Calculation of time:** Time taken by light ray to cross the slab is.

$$T = \frac{\text{Distance}}{\text{Speed}} = \frac{PQ}{v} \qquad \left(PQ = \frac{t}{\cos r}, v = \frac{c}{\mu}\right)$$

Now, $\boxed{T = \frac{\mu t}{c\cos r}}$

### Illustration 44:

Find out: (i) Lateral Shift (ii) Time taken by light ray to cross the slab.




**Solution:**

By Snell's law; $1 \sin 60° = \sqrt{3}\sin r$

$r = 30°$

Lateral shift $d = \frac{t}{\cos r}\sin(i-r) = \frac{10}{\cos 30°}\sin(60°-30°) = \frac{10}{\sqrt{3}}$ cm

Time to cross the slab $= \frac{\mu t}{c\times\cos r} = \frac{\sqrt{3}\times 0.10}{3\times 10^8 \times \cos 30°}$

$= \frac{\sqrt{3}\times 0.10}{3\times 10^8 \times \sqrt{3}/2} = \frac{2}{3}\times 10^{-10}$ sec

## BEGINNER'S BOX-3

**1.** Width of a slab is 6 cm whose $\mu = \frac{3}{2}$. If its rear surface is silvered and object is placed at a distance 28 cm from the front face. Calculate the final position of the image from the silvered surface.

**2.** A light ray is moving from denser (refractive index=μ) to air. If the angle of incidence is half the angle of refraction, find out the angle of refraction.

**3.** Light of wavelength 8000 Å enters from air into water ($\mu_{water} = 4/3$). What is the change in the frequency of light in water?

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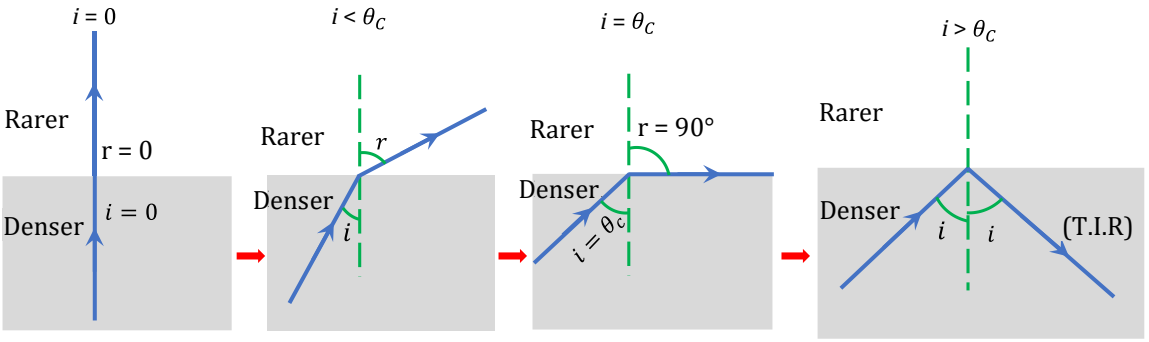
## Total Internal Reflection (T.I.R.)

When light ray travels from denser to rarer medium, it bends away from the normal. If the angle of incidence is increased, the angle of refraction also increases. At a particular value of angle, the refracted ray subtends 90° angle with the normal, this angle of incidence is known as critical angle ($\theta_C$). If angle of incidence increases further, the ray comes back to the same medium. This phenomenon is known as total internal reflection.



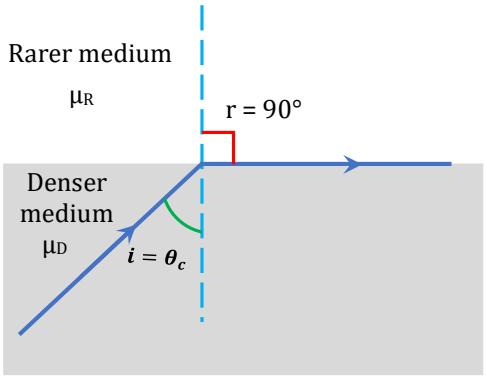

## Calculation of Critical Angle

Using Snell's law

$\mu_D \sin\theta_C = \mu_R \sin 90°$

$$\sin\theta_C = \frac{\mu_R}{\mu_D}$$

$$\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{v_D}{v_R} = \frac{\lambda_D}{\lambda_R}$$




## Conclusion

For T.I.R. light must travel from Denser medium to Rarer medium (for example, glass to air, water to air, glass to water etc.)

**i < $\theta_c$** : Refraction

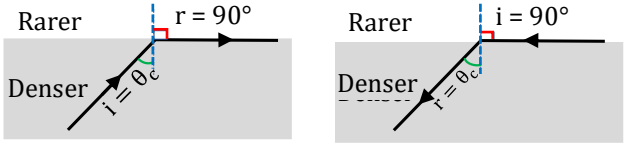
**i = $\theta_c$** : Grazing Emergence (refraction)

**i > $\theta_c$** : TIR

$$\sin\theta_C = \frac{\mu_R}{\mu_D}$$

If rarer medium is air, $\sin\theta_C = \frac{1}{\mu}$

**Note: (1)** From the ray diagram as shown in figure we conclude that in case of refraction maximum angle from normal is $\theta_C$ in denser medium and maximum angle from normal is 90° in rarer medium.




**(2)** In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some loss of intensity as the entire light cannot be reflected or refracted. Due to this reason, images formed by TIR are much brighter those than formed by mirrors or lenses.

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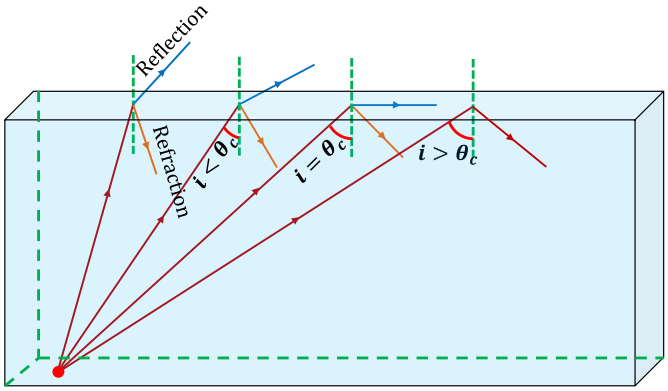



**Illustration 45:**

If the critical angle for a certain medium and vacuum is 30°. Find out the velocity of light in the medium?

**Solution:**

$$\sin\theta_c = \frac{1}{\mu} \Rightarrow \sin 30° = \frac{1}{\mu} \Rightarrow \mu = 2$$

$$v = \frac{c}{\mu} = \frac{3\times10^8}{2} = 1.5\times10^8 \text{ m/s}$$

**Illustration 46:**

Calculate the critical angle for glass-air interface if a ray of light incident on a glass surface is deviated through 15° when angle of incidence is 45°?

**Solution:**

$\delta = i - r \Rightarrow 15° = 45° - r$

$r = 30°$

By snell's law,

$1\sin45° = \mu\sin r$

$$\frac{1}{\sqrt{2}} = \mu\sin30° \Rightarrow \mu = \sqrt{2}$$

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$$

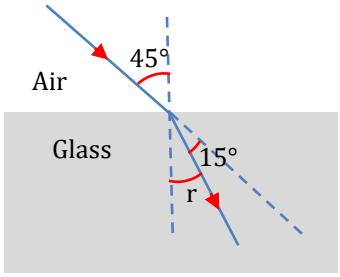
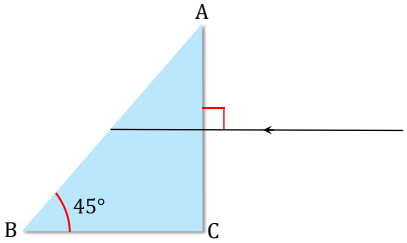



**Illustration 47:**

What should be the range of refractive index of prism so that light ray suffers total internal reflection at surface AB?




**Solution:**

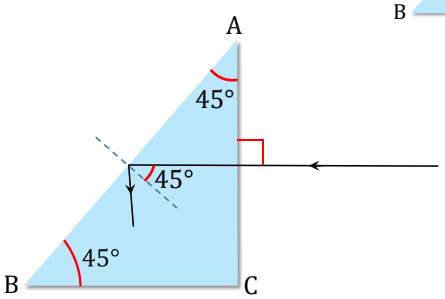
Light ray performs T.I.R. at surface AB

Then $45° > \theta_c$

$\sin45° > \sin\theta_c$

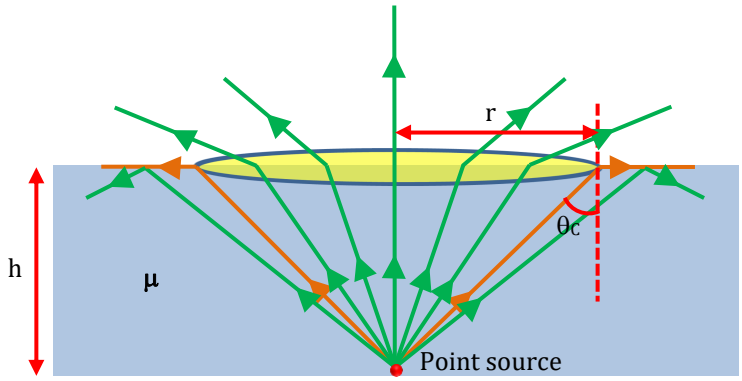
$$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$$

$$\mu > \sqrt{2}$$

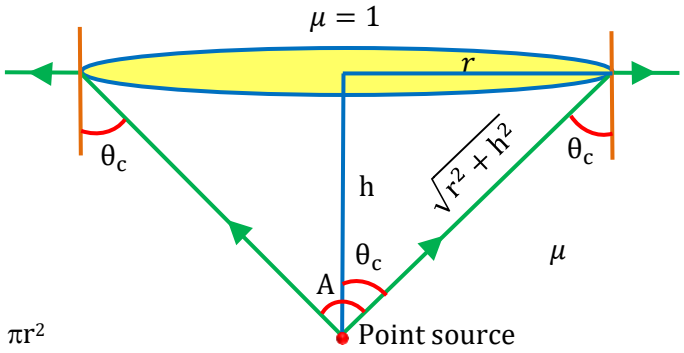


## Circle of illuminance

Ray incident from denser to rarer medium undergoes total internal reflection when their angle of incidence become more than the critical angle, this situation creates a circular region from which light escapes and is called circle of illuminance.




**Calculation:** A point source is situated at the bottom of a tank filled with a liquid of refractive index μ upto h height. It is found that light comes out of liquid surface through a circular portion above the source.




$$\sin\theta_C = \frac{r}{\sqrt{r^2+h^2}} \text{ and}$$

$$\sin\theta_C = \frac{1}{\mu} \Rightarrow \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{r^2+h^2}$$

$$\Rightarrow \mu^2 r^2 = r^2 + h^2 \Rightarrow (\mu^2-1)r^2 = h^2$$

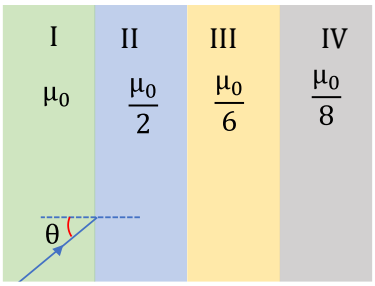
Radius of circular portion, $r = \frac{h}{\sqrt{\mu^2-1}}$ and area = $\pi r^2$

Vertex angle A = $2\theta_C$

For pure water, A = 2 × 49° = 98°

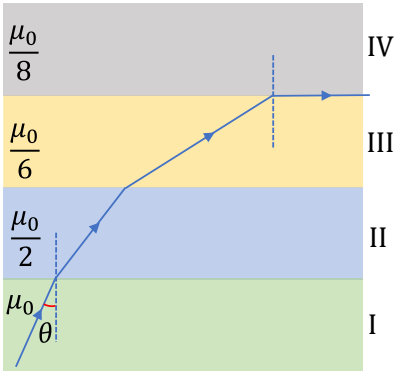
### Illustration 48:

Find out the angle of incidence θ for which light ray just misses entering in region IV?




### Solution:

Apply Snell's Law between I & IV medium,



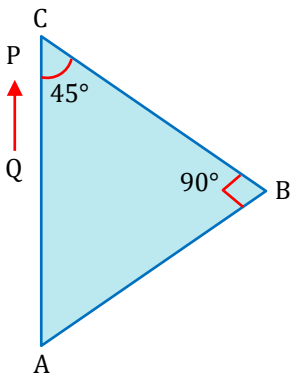

$$\mu_o \sin\theta = \frac{\mu_o}{8}\sin 90°$$

$$\sin\theta = \frac{1}{8}$$

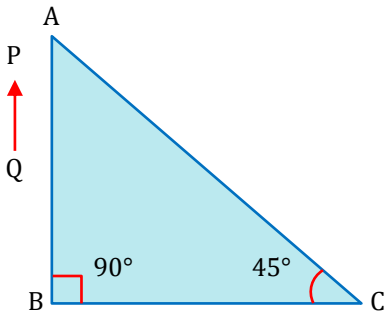
$$\theta = \sin^{-1}\left(\frac{1}{8}\right)$$

### Illustration 49:

An object is placed in front of a right angled prism ABC in two positions (a) and (b) as shown. The prism is made of crown glass with critical angle 41°. Trace the path of rays starting form P and Q as shown in the figures (a) and (b) entering normal to the prism.



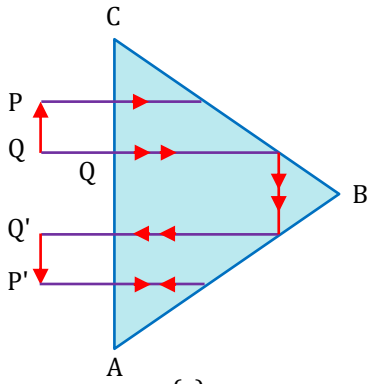

(a)



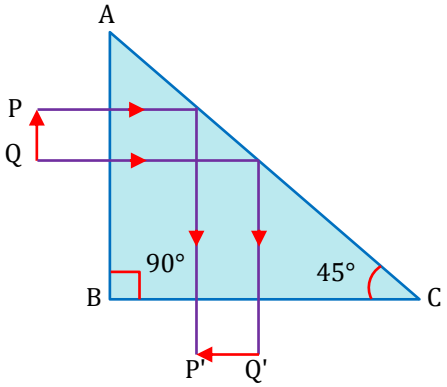

(b)

### Solution:

For the refraction on the inclined surfaces of the prism i = 45° (>C=41°). So, the rays undergo total internal reflection.



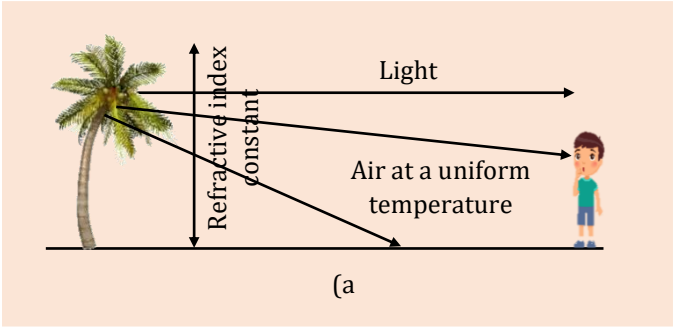

(a)



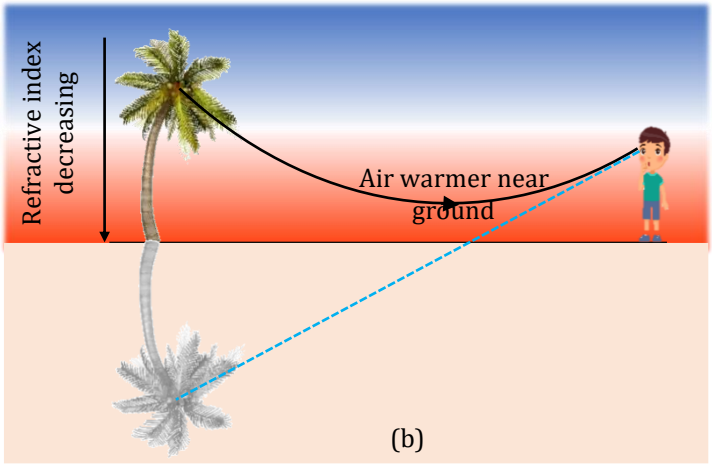

(b)

### Application of TIR

**1. Mirage:** In hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less denser, and has smaller refractive index than the cooler air. If the air currents are small, that is, the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Figure (b). To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called mirage. This type of mirage is especially common in hot deserts. Some of you might have noticed that while moving in a bus or a car during a hot summer day, a distant patch of road, especially on a highway, appears to be wet. But, you do not find any evidence of wetness when you reach that spot. This is also due to mirage.

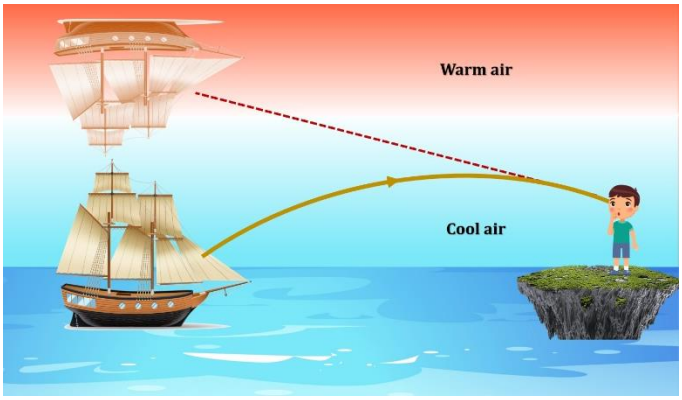


(a



(b)

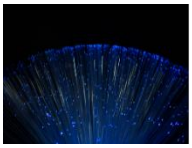
**2. Looming:** Similar to 'mirage' in deserts, 'optical looming' takes place in polar regions due to TIR. Here μ of different air layers decrease with height and so an image of an object is formed in the sky which appears to be suspended in air.




**3. Sparkling of Diamond:** The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_c = 24.4°$. Diamond is cut in such a manner that, once the light enters into it, when it tends come out then $i > \theta_c$. So TIR will take place repeatedly inside it. The light which beams out from a few places entering into the eyes of the observer makes it sparkle.

**4. Optical Fibre:**

- An optical fibre is a cylindrical dielectric waveguide (nonconducting waveguide) that transmits light along its axis through the process of total internal reflection.
- Optical fibre is used as a medium for telecommunication and computer networking



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## Mathematical description in Optical Fibre

Snell's law at air/core interface, $1 \times \sin i = \mu_1 \sin r$

$$\sin r = \frac{\sin i}{\mu_1} \Rightarrow \sin\theta_c = \frac{\mu_2}{\mu_1}$$

For T.I.R. in optical fibre, $90 - r > \theta_c$

Take **sin** on both side, $\sin(90 - r) > \sin\theta_c$

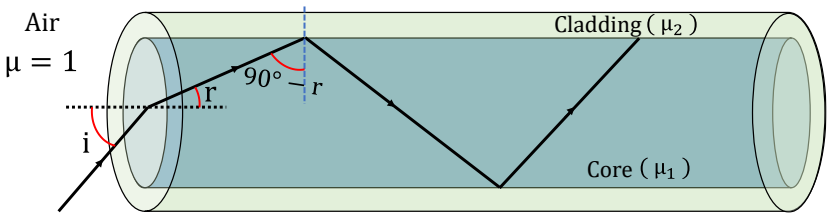
$\cos r > \sin\theta_c$

$$\sqrt{1-\sin^2 r} > \sin\theta_c$$

$$\sqrt{1-\frac{\sin^2 i}{\mu_1^2}} > \frac{\mu_2}{\mu_1}$$

$$\mu_1^2 - \sin^2 i > \mu_2^2$$

$$\boxed{\sin i < \sqrt{\mu_1^2 - \mu_2^2}}$$

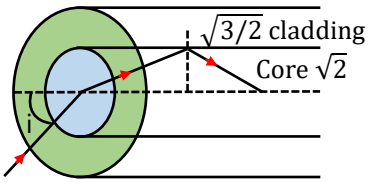



for maximum value,

$\boxed{\sin i = \sqrt{\mu_1^2 - \mu_2^2}}$ Here, i is called maximum acceptance angle.

### Illustration 50:

Figure shows a cross section of a 'light pipe' made of a glass fibre of refractive index $\sqrt{2}$. The outer covering of the pipe is made of a material of refractive index $\sqrt{\frac{3}{2}}$. What is maximum angle of the incident ray with the axis of the pipe for which total internal reflections inside the pipe take place?




### Solution:

$$\sin i < \sqrt{\mu_1^2 - \mu_2^2}$$

$$\sin i < \sqrt{\left(\sqrt{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\sin i < \frac{1}{\sqrt{2}} \Rightarrow i < 45°$$

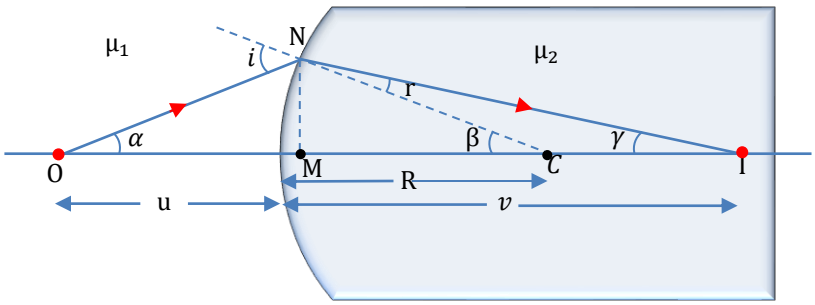
$i_{max} = 45°$

## BEGINNER'S BOX-4

1. If light travels a distance x in time $t_1$ sec in air and 10x distance in time $t_2$ in a certain medium, then find the critical angle of the medium.
2. Calculate the critical angle for glass-air interface if a ray of light incident on a glass surface is deviated through 15° when angle of incidence is 45°.
3. A ray of light travels from denser medium having refractive index $\sqrt{2}$ to air, What should be the angle of incidence for the ray to emerge out ?

## Refraction Through Single Curved Surfaces

### Spherical refraction




By using snell's law, $\mu_1 \sin i = \mu_2 \sin r$

We use paraxial ray so angle i and r are very small

$\sin i \approx i$ & $\sin r \approx r$

$\Rightarrow \mu_1 i = \mu_2 r$ ...(i)

In $\Delta NOC$ $\alpha + \beta = I$ ...(ii)

In $\Delta NIC$ $\gamma + r = \beta$

$r = \beta - \gamma$ ...(iii)

Put the value of i and r in equation (i) from equation (ii) and (iii)

$\mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$ $\quad \tan\alpha = \frac{MN}{OM} \approx \alpha$

$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$ $\quad \tan\beta = \frac{MN}{MC} \approx \beta$

$\mu_1 \frac{MN}{OM} + \mu_2 \frac{MN}{MI} = (\mu_2 - \mu_1)\frac{MN}{MC}$ $\quad \tan\gamma = \frac{MN}{MI} \approx \gamma$

$\frac{\mu_2}{MI} + \frac{\mu_1}{OM} = \frac{\mu_2 - \mu_1}{MC}$

Here OM, MI, MC represents magnitude of distances.

Applying the Cartesian sign

OM = –u, MI = +v, MC = +R

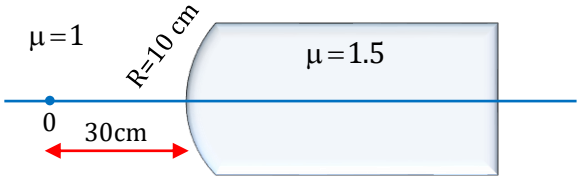
$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

### Important points regarding formula

1. Rays should be paraxial.
2. $\mu_2$ is the medium in which light rays enter after refraction and $\mu_1$ is the medium in which light rays moves before refraction.
3. v , u & R (what ever is given) should be put along with sign.

### Illustration 51:

Find out position and nature of image.




### Solution:

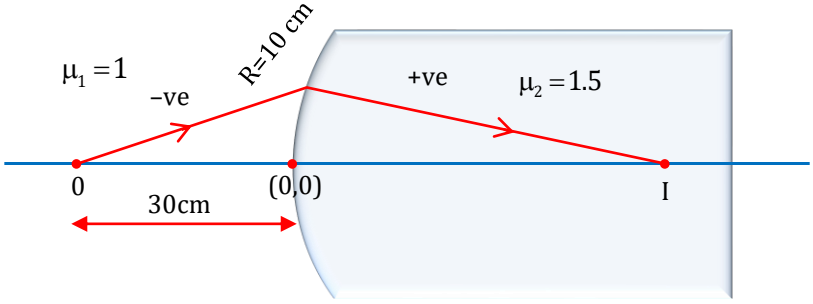
Given, $\mu_2 = 1.5$ ; $\mu_1 = 1$ ; u = –30cm ; R = +10cm

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-30} = \frac{1.5 - 1}{10}$$

$$\frac{3}{2v} = \frac{1}{20} - \frac{1}{30}$$

v = +90 cm (Real Image)

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### Illustration 52:
Find out apparent distance of fish as seen by observer. (Radius of sphere is 10 cm)?

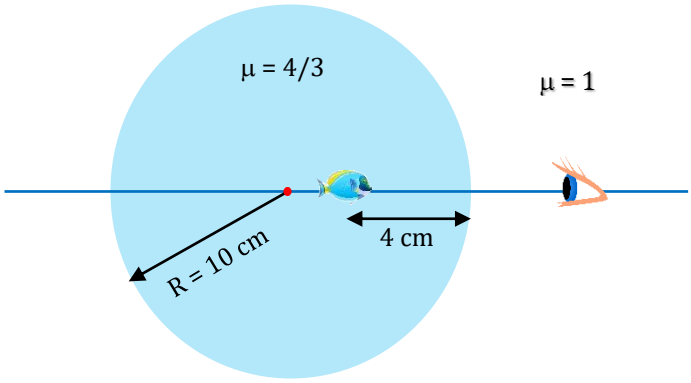
### Solution:
Given, $\mu_2 = 1, \mu_1 = \frac{4}{3}$, u = –4 cm, R = –10 cm

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{4}{3(-4)} = \frac{1 - \frac{4}{3}}{-10}$$

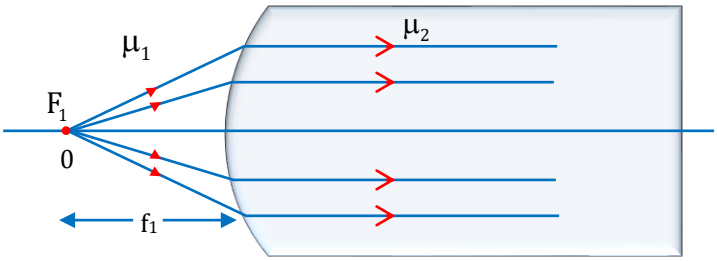
$$\frac{1}{v} + \frac{1}{3} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{3}$$

$$v = \frac{-30}{9} = -3.34\,\text{cm}$$ (virtual image).




### Focal length in spherical surface
A single spherical surface has two principal focal points which are as follows:

**First Focus:** The first principal focus is the point on the axis where an object should be placed so that the image is formed at infinity.



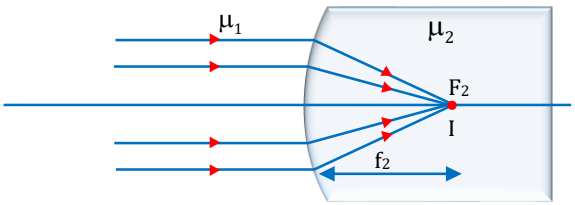

u = $f_1$, v = ∞, then from $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

We get $-\frac{\mu_1}{f_1} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \boxed{f_1 = \frac{-\mu_1 R}{(\mu_2 - \mu_1)}}$

**Second Focus:** The second principal focus is the point where parallel rays get focused.

u = –∞, v = $f_2$, then

$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R}; \boxed{f_2 = \frac{\mu_2 R}{(\mu_2 - \mu_1)}}$$



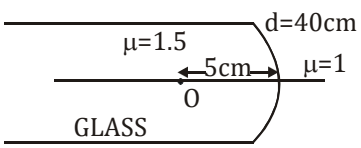

**Ratio of Focal lengths:** $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$

$$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$$

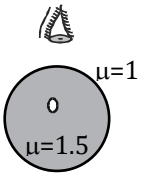
$\boxed{\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}}$ **or** $\boxed{\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0}$

## BEGINNER'S BOX-5

**1.** An object O in glass (μ = 1.5) is situated at a distance of 5 cm from a spherical surface of diameter 40 cm as shown in the figure. Find the distance of the image from the surface.




**2.** There is a small air bubble inside a glass sphere (μ = 1.5) of radius 10 cm. The bubble is 4 cm below the surface and is viewed normally from the outside as shown in figure. Find the apparent depth of the bubble.

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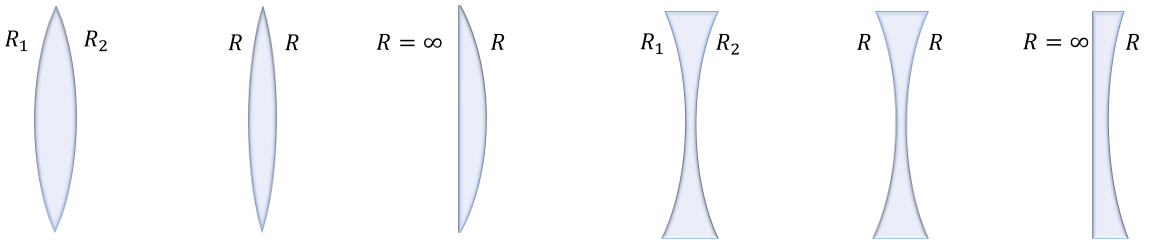
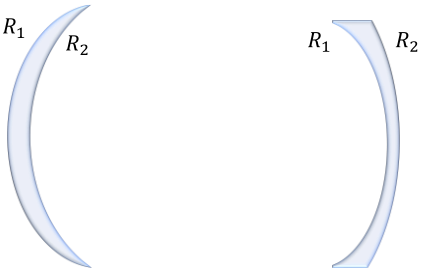
## LENS

- A lens is a portion of a transparent material with two refracting surfaces such that at least one is curved with refractive index of its material being different from that of the surroundings.
- A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.
- However, if the central portion of a lens is thinner than marginal one, it diverges parallel rays passing through it and behaves as divergent or concave lens. This is how we classify convergent and divergent lenses.



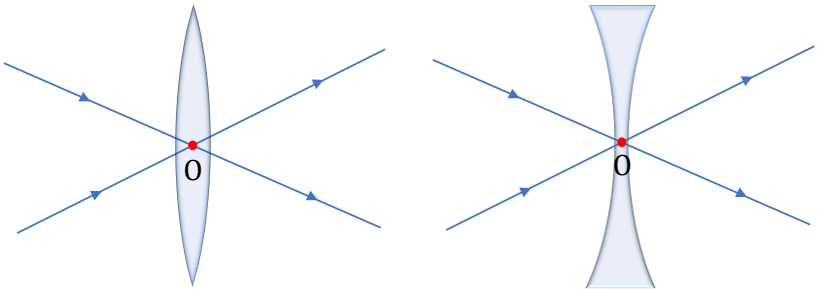

Bi convex Lens | equiconvex Lens | Plano convex Lens | Bi-concave Lens | Equiconcave Lens | Plano concave Lens



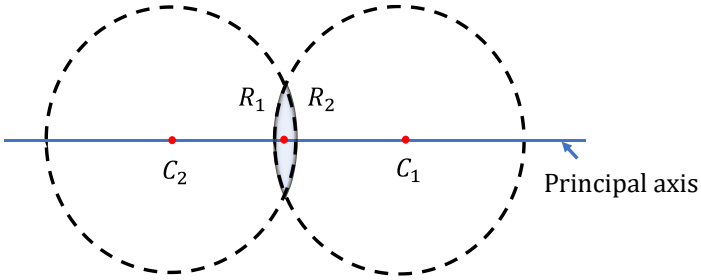

Concavo- convex Lens | Convexo- concave Lens

### Basic terminologies of thin Lens

**Optical Centre:** It is a point O for a given thin Lens, through which any ray passes un-deviated.

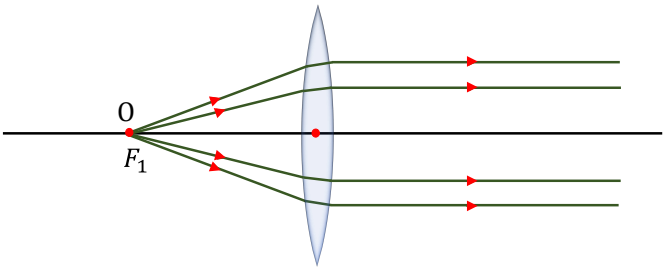



**Principal Axis:** $C_1$ $C_2$ is a line passing through optical Centre and perpendicular to the Lens.

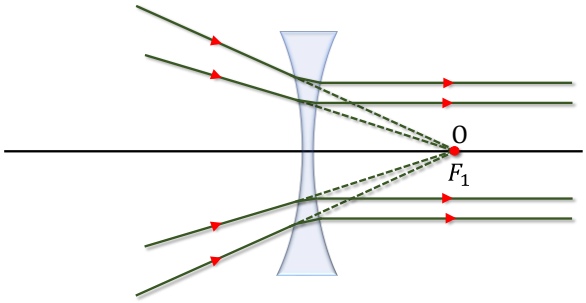


**Principal Focus:** A Lens has two focal points.

**(i) First Focus:** First focal point is an object point on the principal axis corresponding to which the image is formed at infinity.

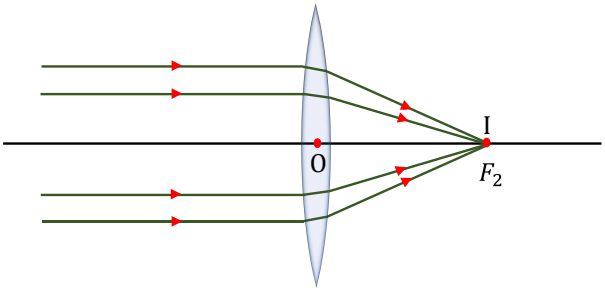



Convex Lens

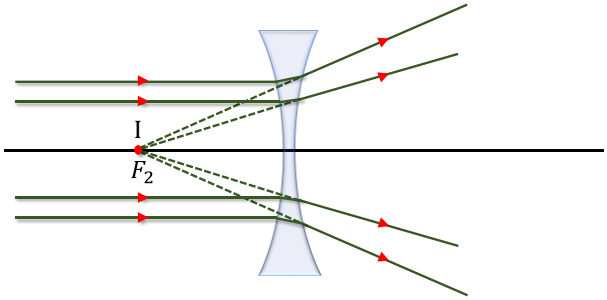



Concave Lens

**(ii) Second Focus:** Second focal point is an image point on the principal axis corresponding to which object lies at infinity



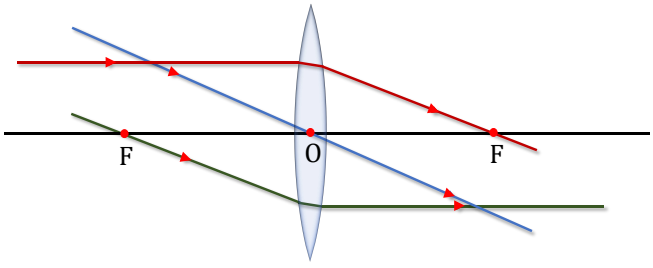

Convex Lens



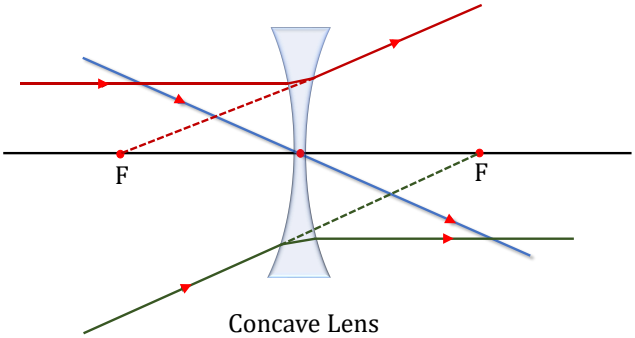

Concave Lens

## Rules for image formation

- A ray passing through optical centre proceeds without deviation through the lens.
- A ray passing through first focus or directed towards it, becomes parallel to the principal axis after refraction from the lens.
- A ray passing parallel to the principal axis passes or appears to pass through F after refraction through the lens.



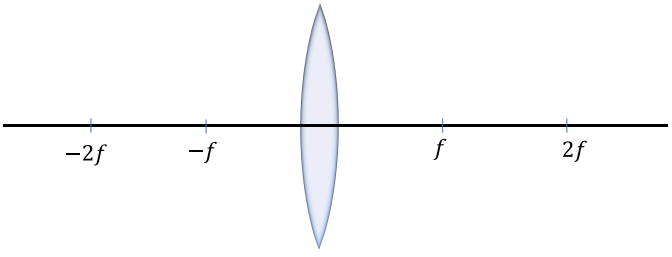

Convex Lens

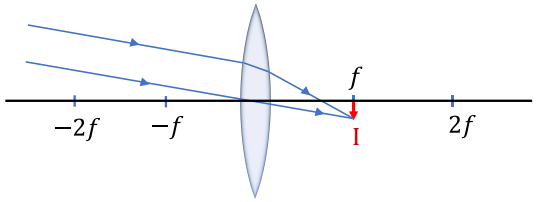
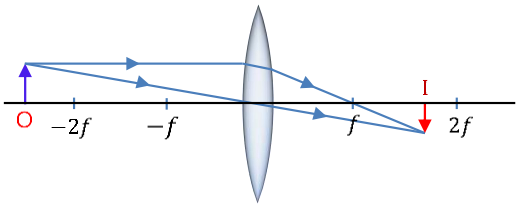
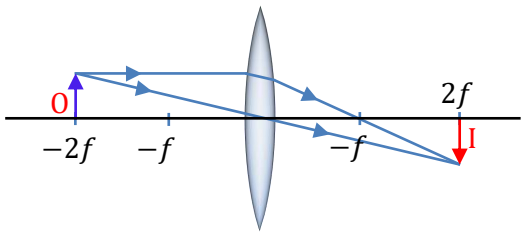
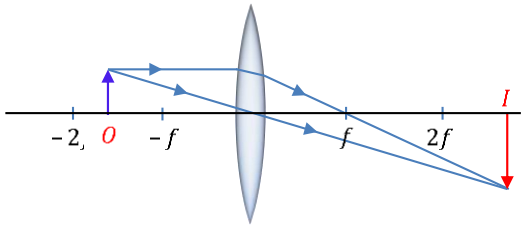
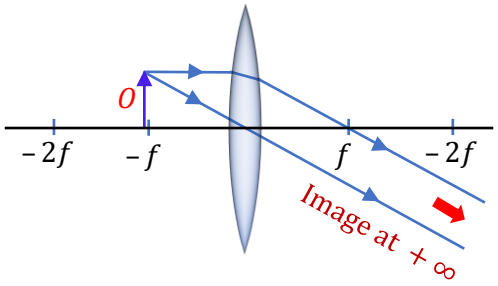
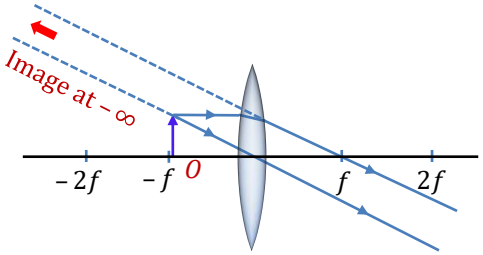



Concave Lens

## Image Formation in Convex and Concave Lens

**Image formation in Convex Lens:**



| S.No. | Position of Object | Position of Image | Nature | Size | Ray Diagram |
|---|---|---|---|---|---|
| **1.** | Place at infinity | Formed at f | Real, Inverted | Highly diminished, |   |
| **2.** | Placed at – ∞ and 2f | Formed in between 2f and f | Real, Inverted | Diminished |   |
| **3.** | Placed at – 2f | Formed at 2f | Real, Inverted | Equal in size |   |
| **4.** | Placed in between –2f and – f | In between 2f and ∞ | Real, Inverted | Enlarged |   |
| **5.** | Placed in between –2f and – f (very nearer to –f) | Formed at +∞ | Real, Inverted | Enlarged |   |
| **6.** | Object is placed in between –f and optical centre (very nearer to –f) | Formed at –∞ | Virtual, erect | Highly enlarged |   |

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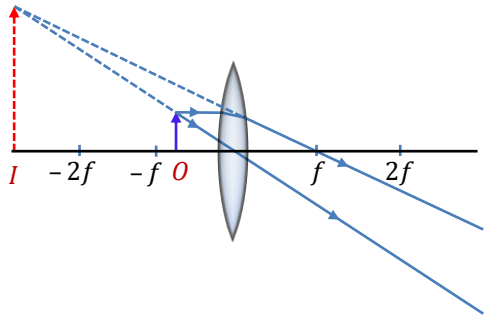
| | | | | | |
|---|---|---|---|---|---|
| **7.** | Object is placed in between –f and optical centre | In between –∞ and optical centre | Virtual, erect | Enlarged |   |

## Image formation in Convex Lens

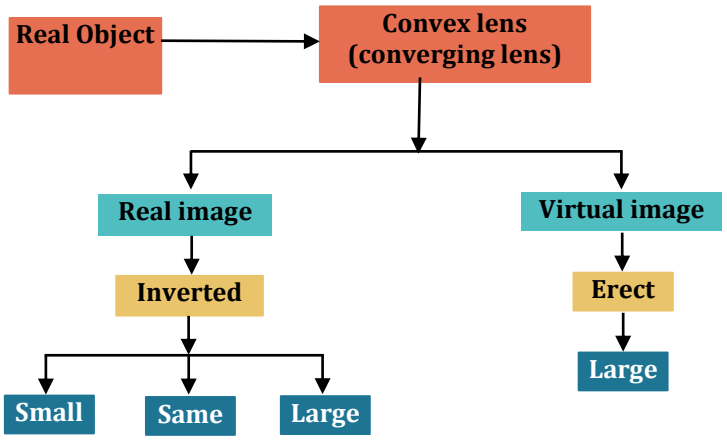



**Image formation in Concave Lens:**

**Object:** Object is placed in between –∞ and optical centre

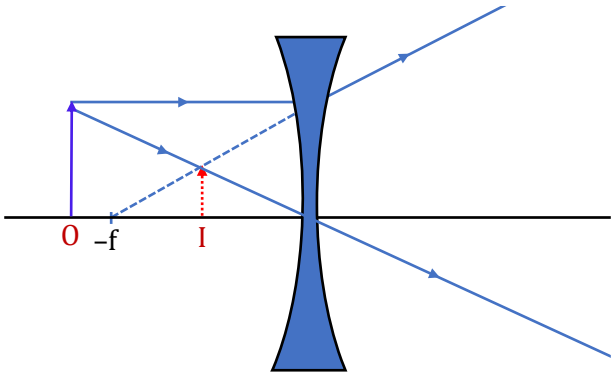



**Image:** in between –f and optical centre, virtual, erect, diminished

Diverging lens always forms virtual image for real object.

### Lens maker's formula & Lens Equations

**Lens maker's formula:** Spherical Refraction from 1st surface,

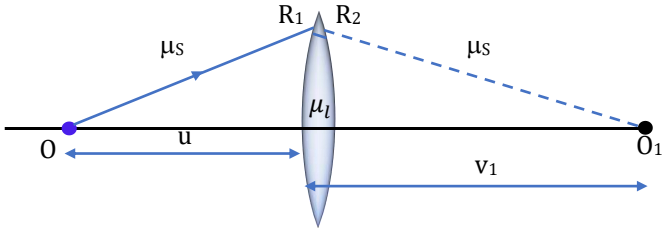
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$\mu_2 = \mu_l$ , $\mu_1 = \mu_S$

$u = u$ , $v = v_1$

$R = R_1$

$$\frac{\mu_l}{v_1}-\frac{\mu_S}{u}=\frac{\mu_l-\mu_S}{R_1} \quad ...(i)$$

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Spherical Refraction from 2nd surface,

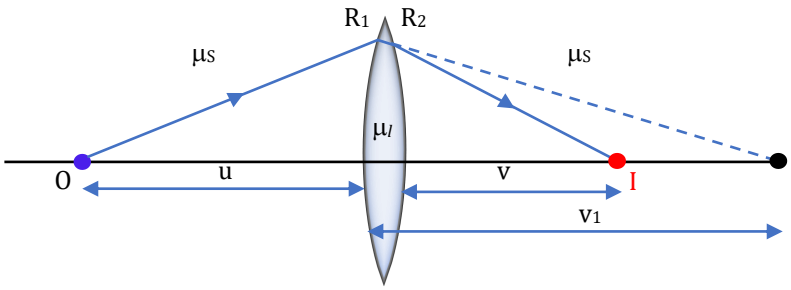
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$\mu_2=\mu_S$ , $\mu_1=\mu_l$

$u=v_1$ , $v=v$

$R=R_2$

$$\frac{\mu_S}{v}-\frac{\mu_l}{v_1}=\frac{\mu_S-\mu_l}{R_2} \quad \text{...(ii)}$$




Add equation (i) & (ii) then,

$$\frac{\mu_S}{v}-\frac{\mu_S}{u}=\frac{\mu_l-\mu_S}{R_1}+\frac{\mu_S-\mu_l}{R_2}$$

$$\frac{\mu_S}{v}-\frac{\mu_S}{u}=(\mu_l-\mu_S)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

If object placed at infinity i.e. $u=-\infty$ then $v=f$

$$\frac{1}{f}-\frac{1}{-\infty}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\boxed{\frac{1}{f}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}$$ Len's maker formula

$\boxed{\frac{1}{v}-\frac{1}{u}=\frac{1}{f}}$ Lens formula. $\Rightarrow$ $\boxed{v=\frac{uf}{u+f}}$

## Important points regarding Lens formula

1. Rays should be paraxial.
2. Lenses should be thin.
3. Medium on both side of the Lens should be same.
4. $R_1$ is the radius of curvature of that surface on which light rays incident initially and $R_2$ is the radius of curvature of that surface on which light rays incident after refraction from the first surface.
5. v, u, $R_1$, $R_2$, f (whatever is given in question) should be put along with sign.

## Sign Convention

To derive the relevant formulae for refraction by spherical lenses, we must first adopt a sign convention for measuring distances. We shall follow the Cartesian sign convention. According to this convention, all distances are measured from the optical centre of the lenses.

- Along the principal axis, distances are measured from the optical centre of the lenses (Optical centre is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite direction negative.
- The distances above the principal axis are taken positive while below it negative.
- Whenever and wherever possible incident light is taken to travel from left to right.

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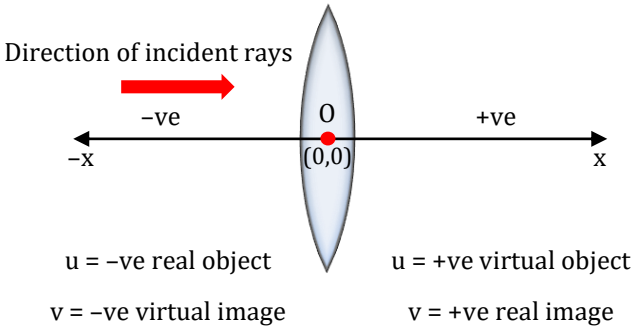
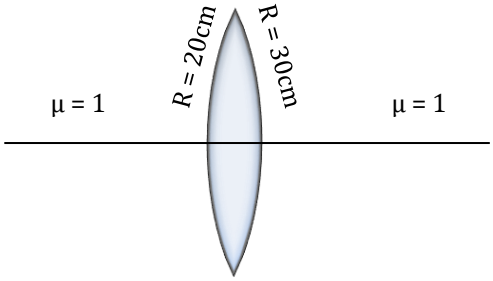



**Note:** (1) For converging Lens (convex Lens in air), focal length is positive.

(2) For diverging Lens (concave Lens in air), focal length is negative.

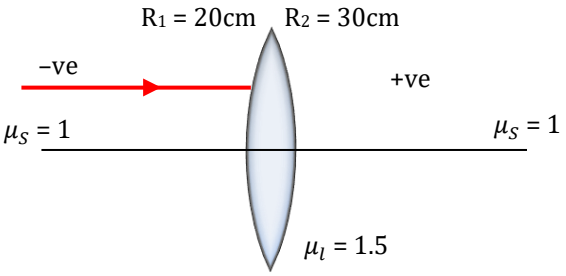
**Illustration 53:**

Find out focal length of the following Lens. ($\mu_\ell = 1.5$)




**Solution:**

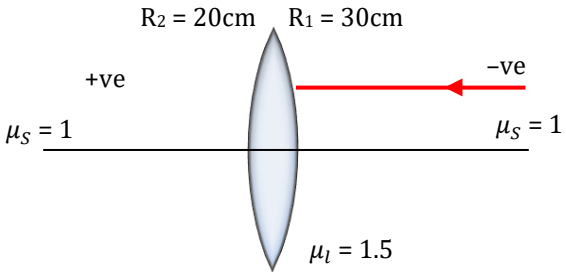
**If Light rays incident from left side**




$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{1.5}{1}-1\right]\left[\frac{1}{20}-\frac{1}{-30}\right]$$

f = 24 cm (converging lens)

**If Light rays incident from right side**



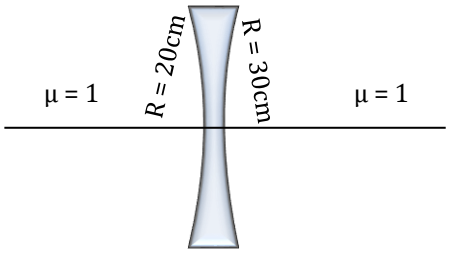

$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{1.5}{1}-1\right]\left[\frac{1}{30}-\frac{1}{-20}\right]$$

f = 24 cm (converging lens)

**Note:** From the above illustration we conclude that focal length of a lens is independent of direction of incident ray.

**Illustration 54:**

Find out focal length of the following Lens. ($\mu_\ell = 1.5$)

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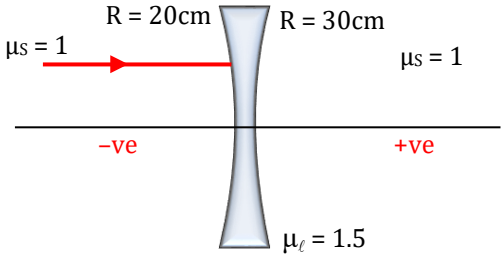
**Solution:**

Given, $\mu_\ell = 1.5, \mu_s = 1, R_1 = -20\text{cm}, R_2 = 30\text{cm}$




$$\frac{1}{f} = \left[\frac{\mu_\ell}{\mu_s} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \left[\frac{1.5}{1} - 1\right]\left[\frac{1}{-20} - \frac{1}{30}\right]$$

f = –24cm (Diverging lens)

**Illustration 55:**

If focal length of convex Lens ($\mu_\ell = 3/2$) in air is 20 cm. Find out focal length of same lens in water. ($\mu_w = 4/3$)

**Solution:**

$$\frac{1}{f} = \left[\frac{\mu_\ell}{\mu_s} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

In Air: $\frac{1}{20} = \left[\frac{3}{2\times1} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ...(i)

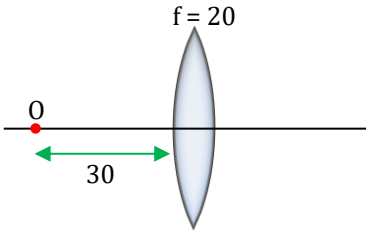
In water: $\frac{1}{f_w} = \left[\frac{3}{2\times4/3} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ...(ii)

by dividing equation (i) by (ii)

$\frac{f_w}{20} = \frac{1/2}{1/8} \Rightarrow f_w = 80$ cm

**Illustration 56:**

Find out position of image?




**Solution:**

Given, u = –30 cm, f = +20 cm, v =?

Using lens formula,

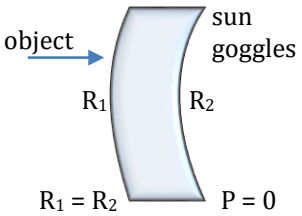
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow v = +60 \text{ cm}$$

**Note:** 1. Sun-glasses or goggles: radii of curvature of two surfaces are equal with centres of curvatures on the same side of the lens.

$R_1 = R_2 = +R$ so, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{R}\right]$

$\Rightarrow \frac{1}{f} = 0 \Rightarrow f = \infty \Rightarrow$ and P = 0 $\left(\therefore P = \frac{1}{f}\right)$

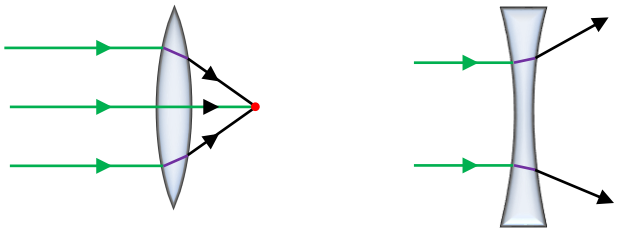
Sun-glasses or goggles have no power.




2. If refractive index of medium < refractive index of lens

If $\mu_s < \mu_L$ then $\frac{\mu_L}{\mu_s} > 1$ or $\left(\frac{\mu_L}{\mu_s} - 1\right) > 0$

Convex lens behaves as a convergent lens. While concave lens behaves as a divergent lens.



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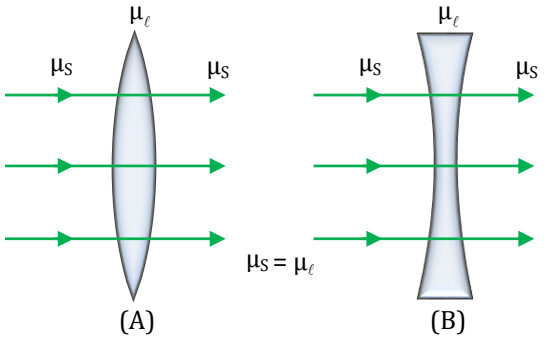
3. Refractive index of medium = Refractive index of lens ($\mu_S = \mu_L$)

$$\frac{1}{f}=\left(\frac{\mu_L}{\mu_S}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right);\ \frac{1}{f}=0\Rightarrow f=\infty\ \&\ P=0$$

Lens will behave as a plane transparent plate




4. Refractive index of surrounding medium > Refractive index of lens

$$\mu_S > \mu_\ell \Rightarrow \frac{\mu_\ell}{\mu_S}<1 \text{ and } \left(\frac{\mu_\ell}{\mu_S}-1\right)<0$$

Convex lens will behave as a divergent lens and concave lens will behave as a convergent lens. An air bubble in water behaves as a concave lens.

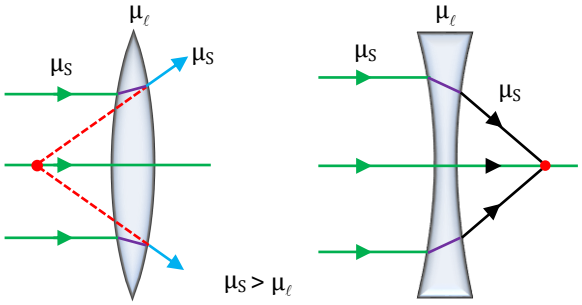



**Illustration 57:**

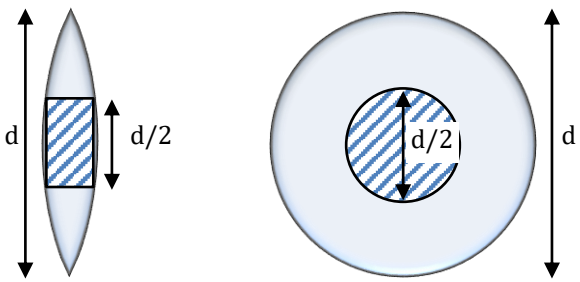
A Lens having focal length f and aperture of diameter d forms an image of intensity I. Aperture of diameter d/2 in central region of Lens is covered by a black paper. Focal length of Lens and intensity of image now will be respectively: -

**Solution:**

Initial area $(A_i)=\pi\left(\frac{d}{2}\right)^2$

Effective area $(A_f)=\pi\left(\frac{d}{2}\right)^2-\pi\left(\frac{d}{4}\right)^2$

$$A_f=\frac{3\pi d^2}{16}=\frac{3}{4}A_i$$




Effective area is $\frac{3}{4}$ times of initial area so image intensity is also $\frac{3I}{4}$.

After covered the lens by black paper no change occurs in radius of curvature of lens, refractive index of lens and surrounding. so focal length of lens remains unchanged.

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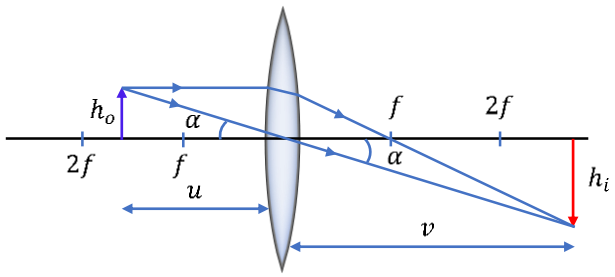
## Transverse or Lateral Magnification ($m_t$)

$$m_t = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_o}$$

$$\tan\alpha = \frac{h_o}{u} = \frac{h_i}{v}$$

$$\frac{h_i}{h_o} = \frac{v}{u}$$

$$\boxed{m_t = \frac{h_i}{h_o} = \frac{v}{u}}$$




**Different forms of Transverse magnification:**

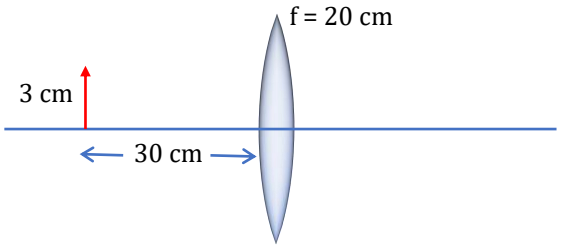
$$m_t = \frac{h_i}{h_o}$$

$$m_t = \frac{h_i}{h_o} = \frac{v}{u}$$

$$m_t = \frac{h_i}{h_o} = \frac{f}{f+u} = \frac{f-v}{f}$$

### Illustration 58:

Find out position, nature and size of image.



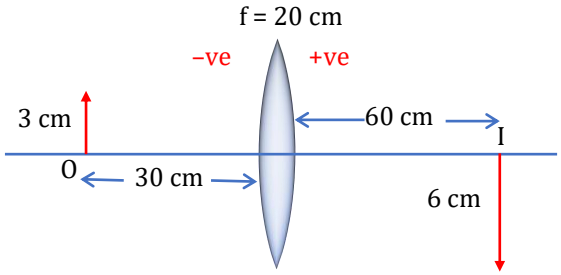

### Solution:

Given, u = –30 cm, $h_o$ = 3 cm, f = 20 cm

$$V = \frac{u\times f}{u+f} = \frac{-30\times 20}{-30+20} \Rightarrow V = 60 \text{ cm (Real)}$$

$$m_t = \frac{h_i}{h_o} = \frac{V}{u} \Rightarrow \frac{h_i}{3} = \frac{60}{-30}$$

$h_i$ = –6 cm (Inverted, Large)




### Illustration 59:

A needle placed 45 cm away from a lens forms an image on a screen placed 90 cm away on the other side of the lens. Identify the type of lens and determine its focal length, what is the size of the image, if the size of the needle is 5 cm?

### Solution:

Here, u = –45 cm, v = 90 cm, f = ?, $h_2$ = ?, $h_1$ = 5 cm,

$$\because \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \therefore \frac{1}{90}+\frac{1}{45}=\frac{1}{f} \Rightarrow \frac{1+2}{90}=\frac{1}{f} \Rightarrow f = 30 \text{ cm}$$

As f is positive, the lens is converging,

$$\because \frac{h_2}{h_1}=\frac{v}{u} \quad \therefore \frac{h_2}{5}=\frac{90}{-45}=-2 \Rightarrow h_2 = -10 \text{ cm.}$$

Minus sign indicates that image is real and inverted.

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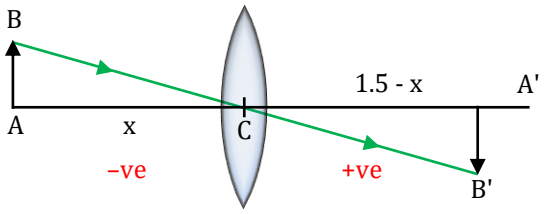
### Illustration 60:

An object is placed at a distant of 1.50 m from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. What is the focal length and the position of the lens?

### Solution:

$m = \frac{h_2}{h_1} = -4$




Let the lens be placed at a distance x from the object.

Then, u = – x, and v = (1.5 – x)

using, $m = \frac{v}{u}$, we get $-4 = \frac{1.5-x}{-x}$

$\Rightarrow x = 0.3$ m

The lens is placed at a distance of 0.3 m from the object (or 1.20 m from the screen)

For focal length, we may use

$m = \frac{f}{f+u} \Rightarrow -4 = \frac{f}{f+(-0.3)} \Rightarrow f = \frac{1.2}{5} = 0.24$ m

### Illustration 61:

An object is placed in front of a convex Lens of focal length 10 cm. Find out position of the object such that we get three times magnified image.

### Solution:

**Case-1:** We get magnified real, inverted image when object lies in between f and 2f

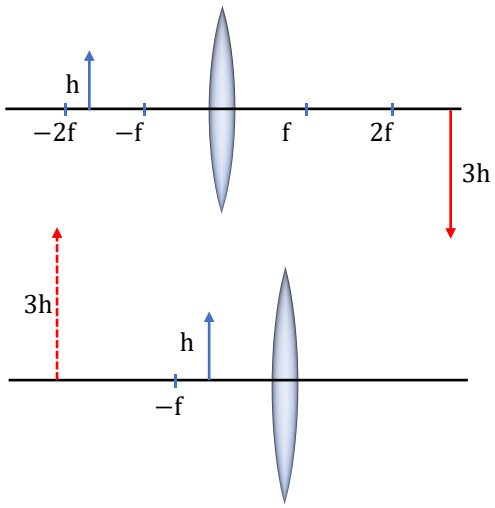
$m_t = \frac{f}{f+u} = -3$

$\frac{10}{10+u} = -3 \Rightarrow u = -\frac{40}{3}$ cm

**Case-2:** We get magnified virtual, erect image when object lies in between f and optical centre.

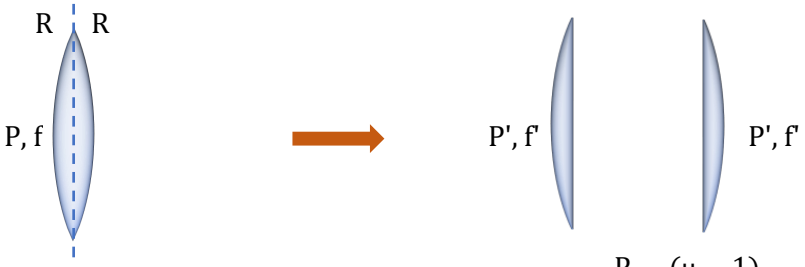
$m_t = \frac{f}{f+u} = 3$

$\frac{10}{10+u} = 3 \Rightarrow u = -\frac{20}{3}$ cm




### Cutting of lens

**Case (1):** When we cut the lens perpendicular to the principal axis : If the equiconvex lens is cut into equal parts by a vertical plane, the focal length of each part will be double the initial value but intensity of image will remain unchanged.




$P = \frac{1}{f} = \frac{2(\mu-1)}{R}$

$f = \frac{R}{2(\mu-1)}$

$P' = \frac{P}{2} = \frac{(\mu-1)}{R}$

$f' = 2f = \frac{R}{(\mu-1)}$

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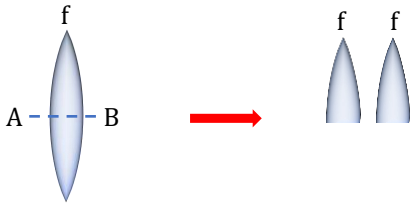
**Case (2):** When we cut the lens parallel to the principal axis: If an equiconvex lens of focal length f is cut into two identical parts by a horizontal plane AB then the focal length of each part will be equal to that of the initial lens; because μ, $R_1$ and $R_2$ will remain unchanged. Only intensity of image will be reduced.




$\therefore$ intensity $I \propto (\text{aperture})^2$

**Illustration 62:**

Two similar planoconvex lenses are combined together in three different ways as shown in the figure. The ratio of the focal length in three cases will be.

**Solution:**

In each case two planoconvex lens are placed close to each other and $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

So, the ratio of focal length in three cases will be $\Rightarrow$ 1 : 1 : 1

**Illustration 63:**

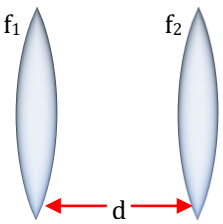
A convex Lens is made up of three different materials as shown in the figure. For a point object placed on its axis, the number of images formed are:

**Solution:**

R.I $\propto$ no. of images

So, number of images = 3

**Note:** **Two thin coaxial lenses are placed at a small separation d** (provided incident rays are parallel to the principal axis).




$$\boxed{\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}}$$

$\Rightarrow$ $\boxed{P = P_1 + P_2 - d\,P_1P_2}$

Use proper sign convention when solving numerical.

**Illustration 64:**

A convex lens of focal length 25 cm and a concave lens of focal length 20 cm are mounted coaxially separated by a distance d cm. If the power of the combination is zero, d is equal to :

**Solution:**

$P_1=\frac{1}{f_1}=\frac{100}{25}=4D$ and $P_2=\frac{1}{f_2}=\frac{100}{-20}=-5D$

$P=P_1+P_2-d\ P_1P_2=0$

4 – 5 + d (4) (5) = 0

d = 0.05 m = 5 cm

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## BEGINNER'S BOX-6

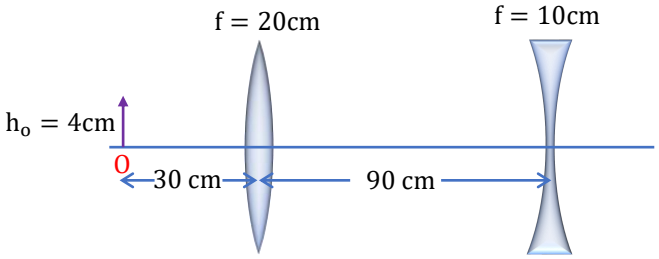
1. An object is placed at the distance of 30 cm in front of a convex lens of focal length 10 cm. Find the position of the image, its nature and magnification.
2. A planoconvex lens has a focal length of 30 cm and an index of refraction 1.5. Find the radius of the convex surface.
3. A biconvex lens (μ = 1.5) of focal length 0.2 m acts as a divergent lens of power 1D when immersed in a liquid. Find the refractive index of the liquid.
4. Two thin converging lenses of focal lengths 20 cm and 40 cm are placed in contact. Find the effective power of the combination.
5. An object placed 20 cm in front of a convex lens has its image 40 cm behind the lens. Find the power of the lens.
6. A lens shown in figure is made of two different materials. A point object is placed on its axis. How many images will be formed.

7. A convex lens of focal length f produces a real image of size is $\frac{1}{n}$ times the size of the object. Find the position of the object.

## Combination of Lens

### Illustration 65:

Find out height of final image formed.




**Solution:**

1st Lens (Convex lens)

u= –30 cm

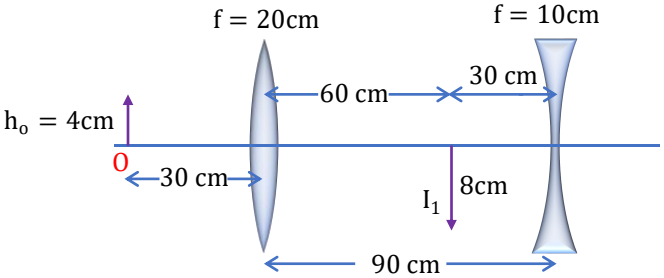
f = 20 cm

$h_o$ = 4 cm

$$V = \frac{u \times f}{u + f} = \frac{-30 \times 20}{-30 + 20} = 60\,cm \quad (I_1 \text{ image})$$

$$h_i = \frac{V}{u} h_o = \frac{60}{-30} \times 4 = -8\,cm$$

Image of convex lens will behave as an object for concave lens (2nd lens).




Now for 2nd lens (concave lens)

u = –30 cm, f = –10 cm, $h_o$ = –8 cm

$$V = \frac{u \times f}{u + f} = \frac{-30 \times -10}{-30 - 10} = -\frac{300}{40} = -7.5\,cm$$

$$h_i = \frac{V}{u} h_o = \frac{-7.5}{-30} \times (-8) = -2\,cm$$

So final image formed at a distance of 7.5 cm from the concave lens and height of final image is 2 cm.

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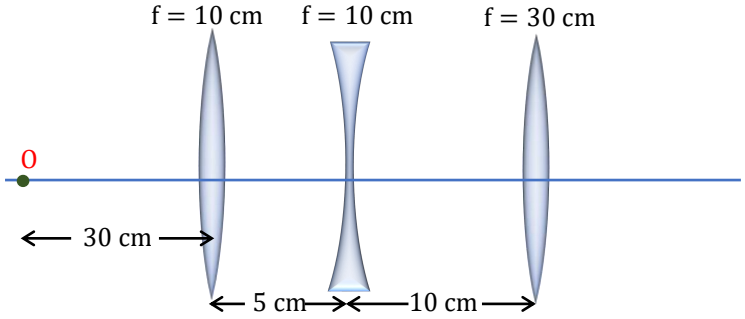
**Illustration 66:**

Find the position of the image formed by the Lens combination given in the figure:




**Solution:**

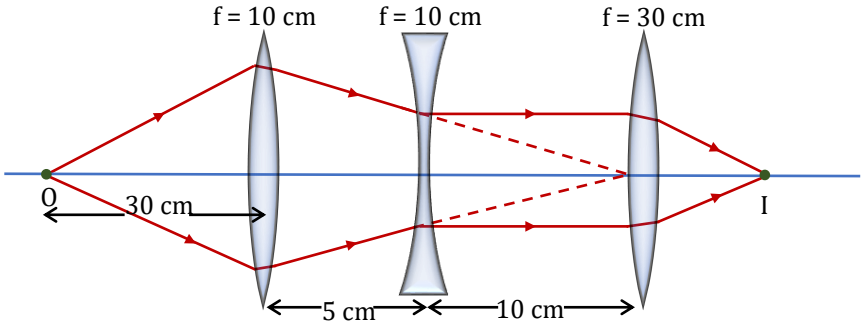



Image formed by the first lens, $\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}\Rightarrow \frac{1}{v_1}-\frac{1}{-30}=\frac{1}{10}\Rightarrow v_1 = 15$ cm

The image formed by the first lens serves as the object for the second. This is at a distance of (15 – 5) cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens. $\frac{1}{V_2}-\frac{1}{10}=\frac{1}{-10}\Rightarrow V_2 = \infty$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens. $\frac{1}{V_3}-\frac{1}{u_3}=\frac{1}{f_3}\Rightarrow \frac{1}{v_3}=\frac{1}{\infty}+\frac{1}{30}\Rightarrow V_3 = 30$ cm

The final image is formed 30 cm to the right of the third lens.

**Velocity of Image in Lens**

**Case (1):** When the object is moving along to principal axis of the Lens.

From equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Differentiate with respect to time,

$$-\frac{1}{v^2}\frac{dv}{dt}-\frac{-1}{u^2}\frac{du}{dt}=0$$

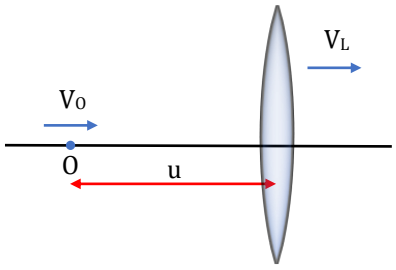
$$-\frac{1}{v^2}\frac{dv}{dt}=\frac{-1}{u^2}\frac{du}{dt}$$

$$\frac{dv}{dt}=+\frac{v^2}{u^2}\frac{du}{dt}$$

$\boxed{\vec{V}_{IL}=m_t^2\vec{V}_{OL}}$

$V_{IL}$ = velocity of image with respect to lens

$V_{OL}$ = velocity of object with respect to lens




All velocities are instantaneous.

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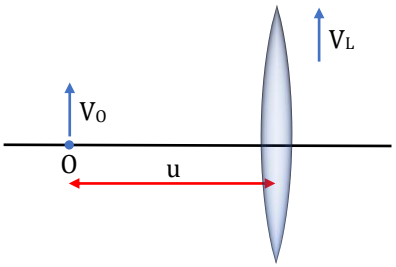
**Case (2) :** When the object is moving perpendicular to the principal axis of the Lens.

From equation $m_t = \frac{h_i}{h_o} = \frac{v}{u}$

$$h_i = \frac{v}{u} h_o$$



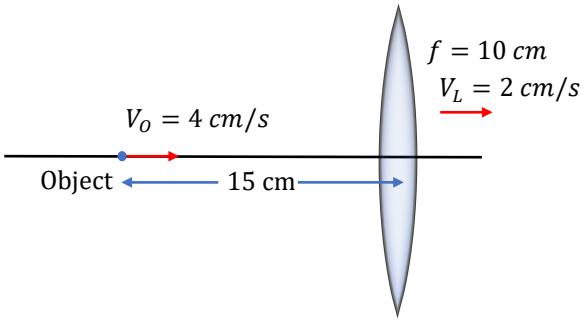

Differentiate with respect to time,

$$\frac{dh_i}{dt} = \frac{v}{u}\frac{dh_o}{dt}$$

$\boxed{\vec{V}_{IL} = m_t \vec{V}_{OL}}$ All velocities are instantaneous.

### Illustration 67:

Find out velocity of image.




### Solution:

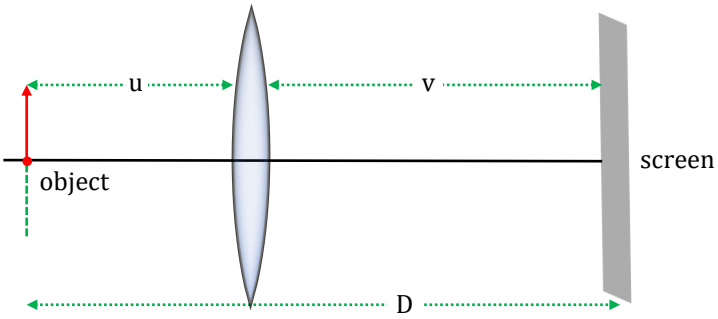
$$m_t = \frac{f}{f+u} = \frac{10}{10-15} = -2$$

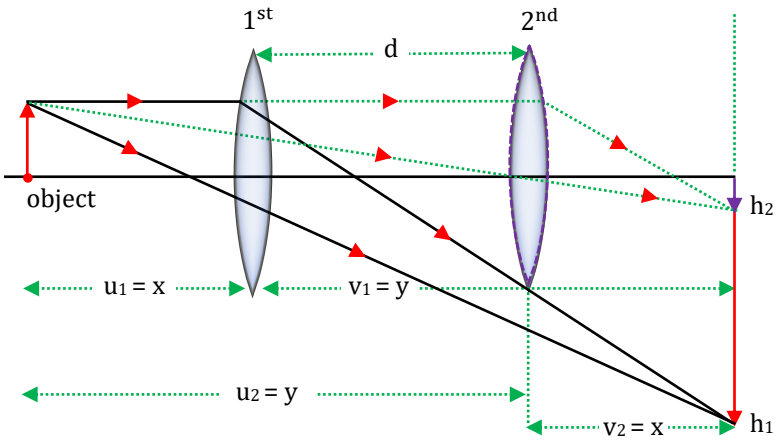
$$\vec{V}_{IL} = m_t^2 \vec{V}_{OL} \Rightarrow \vec{V}_I - \vec{V}_L = m_t^2\left(\vec{V}_0 - \vec{V}_L\right) \Rightarrow \vec{V}_I - 2 = 4(4-2)$$

$$\vec{V}_I = 10\ \text{cm/s}$$

### Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart. If D > 4f there are two positions of lens corresponding to which a sharp image of the object is formed on the screen.



| Calculation of focal length | Calculation of height of object |
|---|---|
| $x + y = D$ ...(i)<br>$y - x = d$ ...(ii)<br>From equation (i) & (ii)<br>$y = \frac{D+d}{2}$ and $x = \frac{D-d}{2}$<br>When Lens is in 1st position<br>$u = -x, v = y$<br>From Lens formula,<br>$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{y} - \frac{1}{-x} = \frac{1}{f}$<br>$\frac{2}{D+d} + \frac{2}{D-d} = \frac{1}{f} \Rightarrow \boxed{f = \frac{D^2 - d^2}{4D}}$ | When Lens in 1st position<br>$m_1 = \frac{y}{x} = \frac{h_1}{h_o}$<br>When Lens is in 2nd position<br>$m_2 = \frac{x}{y} = \frac{h_2}{h_o}$<br>Multiply both equation,<br>$m_1 \times m_2 = 1 = \frac{h_1 h_2}{h_o^2}$<br>$\boxed{h_o = \sqrt{h_1 h_2}}$ |

**Note:**

(1) If D > 4f Two position of lens could be found to form the real image of the source on the screen.
(2) If D = 4f One position of lens could be found to form the real image of the source on the screen.
(3) If D < 4f No position of lens could be found to form the real image of the source on the screen.

So, for image form on the screen, $D \geq 4f \Rightarrow \boxed{f \leq \frac{D}{4}}$

**Conclusion**

(1) $f = \frac{D^2 - d^2}{4D}$ (2) $m_1 \times m_2 = 1$ (3) $h_o = \sqrt{h_1 \times h_2}$ (4) $|m_1 - m_2| = \frac{d}{f}$

(5) $D \geq 4f$ where, d is the displacement of Lens

(6) $f_{max} = \frac{D}{4}$ D is the distance between object and screen

**Illustration 68:**

A screen is placed a distance 40 cm away from an illuminated object. A converging lens is placed between the source and the screen and it is attempted to form the image of the source on the screen. If no position could be found then find out the focal length of the lens.

**Solution:**

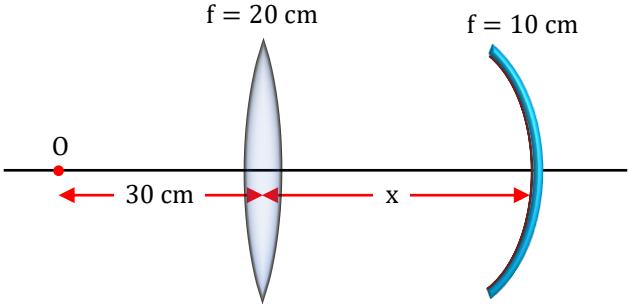
Given, D = 40 cm

If no position of lens could be found to form the image i.e. $D < 4f \Rightarrow 40 < 4f \Rightarrow f > 10$ cm

## Combination of Lens & Mirror

**Illustration 69:**

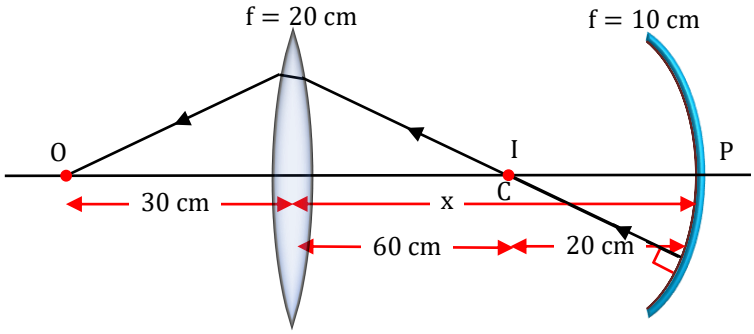
Find out the value of **x** such that light ray retraced its path after reflection from mirror.



**Solution:**

Given for lens, u = –30 cm, f = 20 cm

$v = \frac{u \times f}{u + f} = \frac{-30 \times 20}{-30 + 20} = 60$ cm

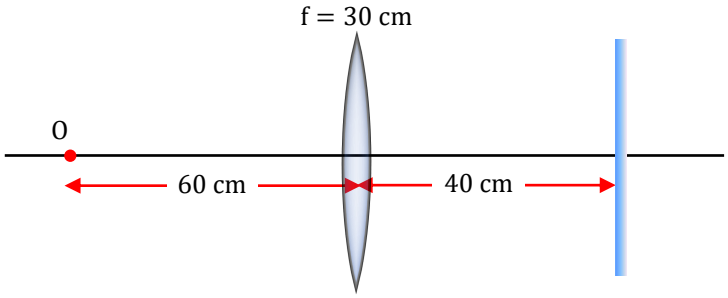



Concave mirror is to be placed to the right in such away that the light rays fall on the mirror perpendicularly (means pass through center of curvature of mirror). Thus, the the image formed by the lens should be at the centre of curvature of the mirror. So, IP = 2f = 2×10 = 20 cm.

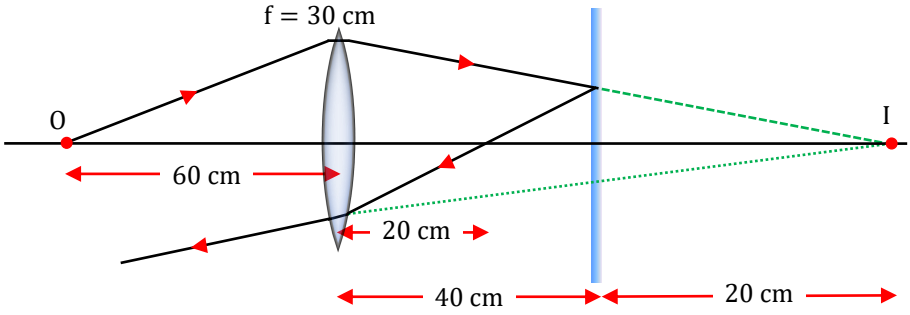
Distance between lens and mirror is x = 60 + 20 = 80 cm

**Illustration 70:**

A point object is placed at a distance of 60 cm from a convex lens of focal length 30 cm. If a plane mirror were put perpendicular to the principal axis of the lens and at a distance of 40 cm from it, the final image would be formed at a distance of:




**Solution:**




$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v_1} - \frac{1}{-60} = \frac{1}{30}$

$v_1 = 60$ cm

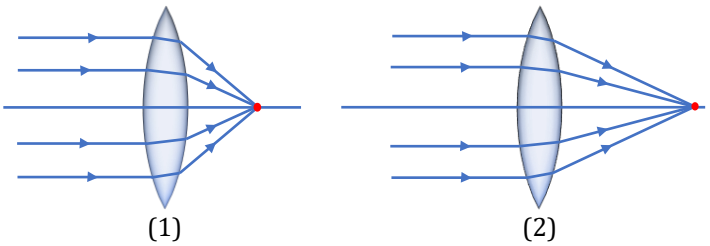
$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v_2} - \frac{1}{-20} = \frac{1}{30} \Rightarrow v_2 = -60$cm (from lens)

**Optical Power (Lens and Mirror)**

Optical power is the degree to which a lens, mirror, or other optical system converges or diverges light. Optical power also referred to as dioptric power, refractive power, focusing power, or convergence power.

**Unit of optical power is diopter ($m^{-1}$).**

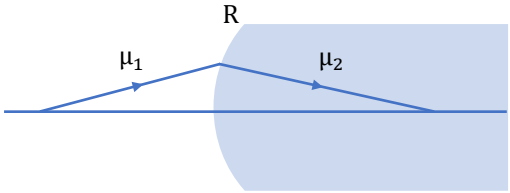


Optical power of 1st lens is greater than the optical power of 2nd lens.

- **Power of any converging optical system is positive.**
  Example : Convex lens in air, concave mirror.
- **Power of any diverging optical system is negative.**
  Example : Concave lens in air, convex mirror.

## Optical power of different optical system

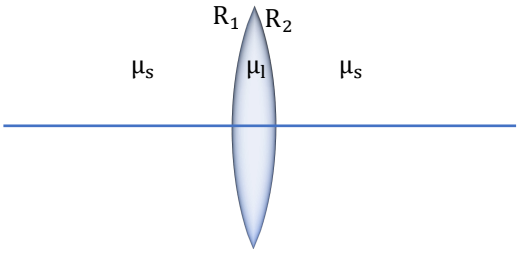
(1) Spherical surface:




$$\boxed{P = \frac{\mu_2 - \mu_1}{R}}$$

- Put the value of R with sign.
- $\mu_2$ is the medium in which light rays enter after refraction and $\mu_1$ is the medium in which light rays moves before refraction.

(2) Thin lens:



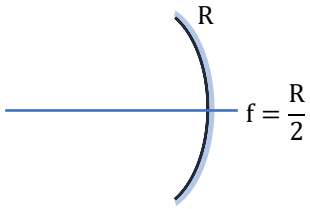

$$\boxed{P = \frac{\mu_s}{f}}$$

$$\boxed{\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

$$P = \mu_S\left(\frac{\mu_\ell}{\mu_S} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boxed{P = (\mu_\ell - \mu_S)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

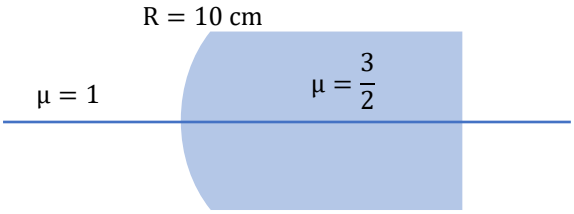
(3) Spherical Mirror:




$$\boxed{P = -\frac{1}{f}}$$

### Illustration 71:

Find out optical power of the spherical surface.

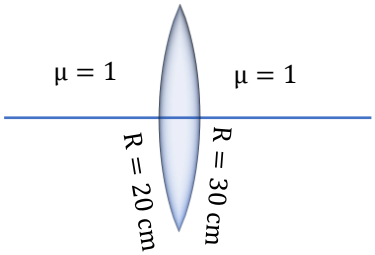



**Solution:**

$$P = \frac{\mu_2 - \mu_1}{R} = \left[\frac{3/2 - 1}{10}\right] \times 100 = 5D \text{ (converging system).}$$

### Illustration 72:

Find out optical power of the lens ($\mu_l = 1.5$).



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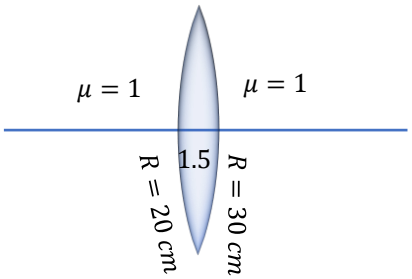
**Solution:**

**First method:** $\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

$\frac{1}{f}=\left[\frac{1.5}{1}-1\right]\left[\frac{1}{20}-\left(-\frac{1}{30}\right)\right]$

$f = 24$ cm

$P = \frac{1}{f}=\frac{100}{24}=\frac{25}{6}D$




**Second method:**

Through two spherical refraction

$P = \left[\frac{1.5-1}{20}+\frac{1-1.5}{-30}\right]\times 100$

$P = \frac{25}{6}D$

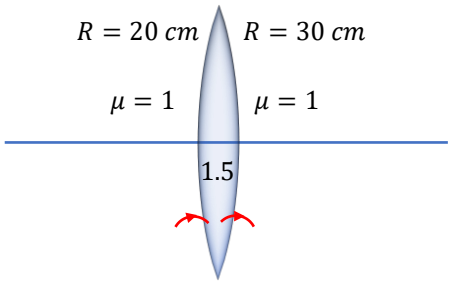



**Illustration 73:**

Find out optical power of equi-concave lens of refractive index μ and radius of curvature R.

**Solution:**

$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{\mu}{1}-1\right]\left[\frac{1}{-R}-\frac{1}{R}\right]$

$f=-\frac{R}{2(\mu-1)}\Rightarrow \boxed{P=-\frac{2(\mu-1)}{R}}$ (Learn directly), Power is –ve, so lens is diverging.

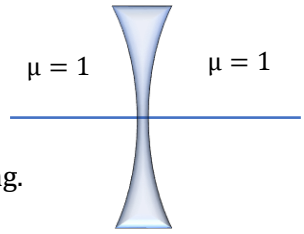



**Illustration 74:**

Find out optical power of equiconvex lens of refractive index μ and radius of curvature R.

**Solution:**

$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{\mu}{1}-1\right]\left[\frac{1}{R}-\frac{1}{-R}\right]$

$f=\frac{R}{2(\mu-1)}$

$\boxed{P=\frac{2(\mu-1)}{R}}$ (Learn directly), Power is +ve, so lens is converging.

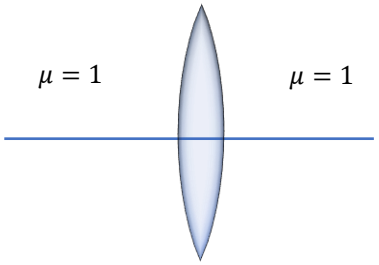



**Illustration 75:**

Power of a convex lens (μ = 3/2) in air is 10D, now we immersed the lens in water (μ = 4/3) then find out the power of lens in water?

**Solution:**

$P=\left[\mu_\ell-\mu_s\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

In medium-1: $P_1=\left[\mu_\ell-\mu_{s_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(i)

In medium-2: $P_2=\left[\mu_\ell-\mu_{s_2}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(ii)

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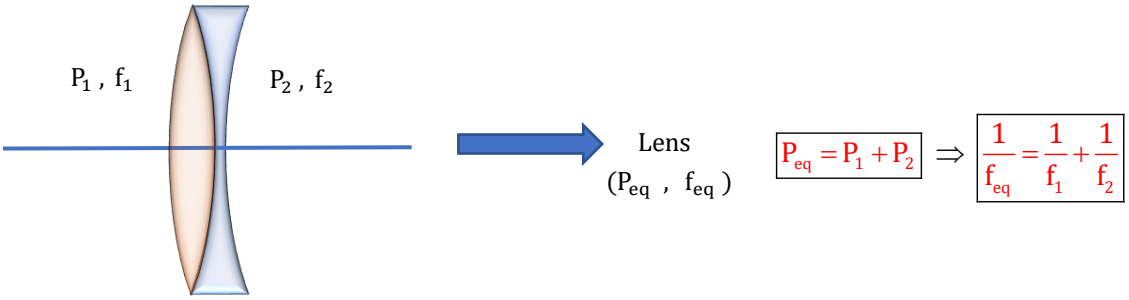
By dividing equation (i) by (ii), $\boxed{\frac{P_1}{P_2}=\left[\frac{\mu_\ell-\mu_{s_1}}{\mu_\ell-\mu_{s_2}}\right]}$

Now medium-1 is air and medium-2 is water, $\boxed{\frac{P_a}{P_w}=\left[\frac{\mu_\ell-\mu_a}{\mu_\ell-\mu_w}\right]}$

$\frac{10}{P_w}=\frac{3/2-1}{3/2-4/3}\Rightarrow P_w=\frac{10}{3}D$ ; Power of convex lens in water is less than that of air.

## Combination of lenses

Two or more than two lens are placed in contact with each other.

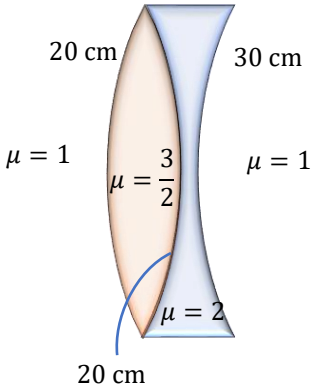



$\boxed{P_{eq}=P_1+P_2}\Rightarrow\boxed{\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}}$

If $P_{eq}$ is +ve then **converging lens** is formed.
If $P_{eq}$ is –ve then **diverging lens** is formed.
If $P_{eq}$ is zero then **plane surface** is formed.
Use sign convention while solving numerical.

### Illustration 76:

Find out $P_{eq}$ & $f_{eq}$?




### Solution:

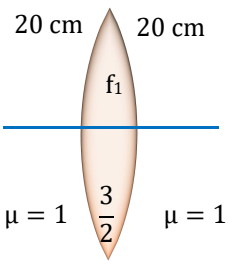
$$\frac{1}{f_1}=\left[\frac{3}{2}-1\right]\left[\frac{1}{20}+\frac{1}{20}\right]=\frac{1}{20}$$

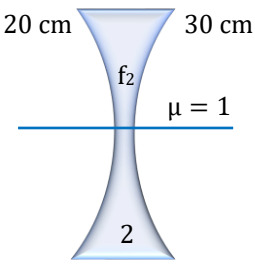
$$\frac{1}{f_2}=[2-1]\left[-\frac{1}{20}-\frac{1}{30}\right]$$

$$\frac{1}{f_2}=-\frac{1}{12}\Rightarrow\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{f_{eq}}=\frac{1}{20}-\frac{1}{12}\Rightarrow f_{eq}=-30cm$$

$$P_{eq}=\frac{\mu_s}{f_{eq}}=-\frac{100}{30}=-\frac{10}{3}D$$

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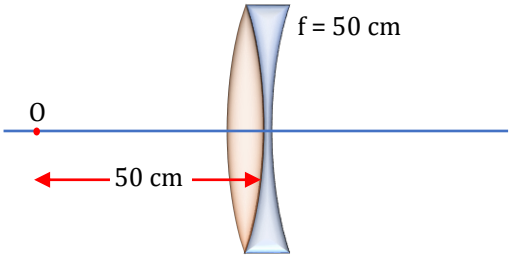
### Illustration 77:

Lens of power 3D and –5D are combined to form a compound lens. An object is placed at a distance of 50 cm from the lens. Calculate the position of its image




**Solution:**

Total power $P_T = 3D - 5D = -2D$

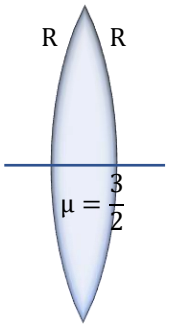
$f = \frac{1}{P} = -\frac{100}{2} = -50\,cm$ (Diverging lens)

Now, $v = \frac{u \times f}{u + f} = \frac{-50 \times -50}{-50 - 50} = -25cm$

### Illustration 78:

Two identical glass ($\mu = 3/2$) equiconvex lens of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu = 4/3$). Find out the focal length of the combination?

**Solution:**




$$f = \frac{R}{2(\mu - 1)}$$

$$f = \frac{R}{2\left(\frac{3}{2} - 1\right)} = R$$

$f_1 = f$

$$\frac{1}{f_2} = \left[\frac{4}{3} - 1\right]\left[-\frac{2}{R}\right] = -\frac{3R}{2} \Rightarrow f_2 = -\frac{3f}{2}$$

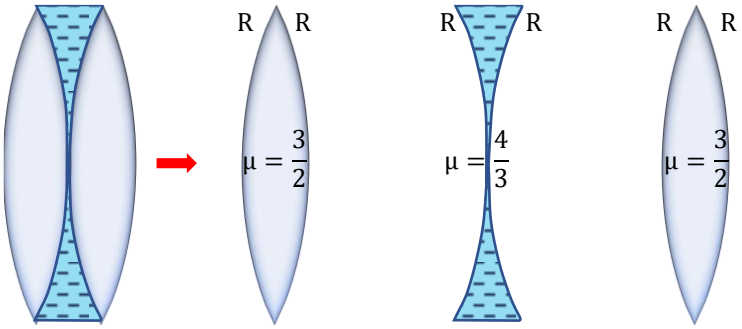
and $f_3 = f$

So equivalent focal length,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\frac{1}{f_{eq}} = \frac{1}{f} - \frac{2}{3f} + \frac{1}{f} = \frac{4}{3f}$$

$$f_{eq} = \frac{3f}{4}$$




## Silvering of Lens

When several lenses or mirrors are used, the image formation is considered one after another in sequences of steps, the image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens acts as an object for the third, and so on, the total magnification in such situations will be given by,

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times ..... \Rightarrow m = m_1 \times m_2 \times .....$$

Power of Lens [in air] $P_L = \frac{1}{f_L}$ ; Converging lens $P_L$ = +ve ; Diverging lens $P_L$ = –ve

Power of mirror $P_m = -\frac{1}{f_m}$; Convex mirror $P_M$ = –ve ; Concave mirror $P_M$ = +ve

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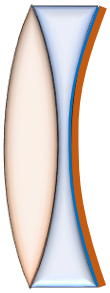


Mirror

$(P_{eq}, f_{eq})$

$$P_{eq} = 2P_l + P_m$$

If $P_{eq} \to$ + ve $\to$ concave mirror

If $P_{eq} \to$ – ve $\to$ convex mirror

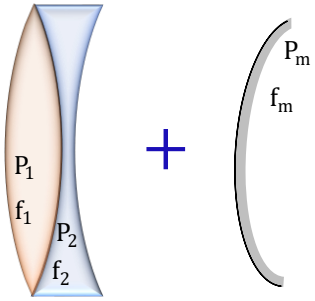
If $P_{eq} \to 0 \to$ plane mirror

Calculation of $f_{eq}$ :

$P_{eq} = 2P_l + P_m \Rightarrow P_{eq} = 2(P_1 + P_2) + P_m$

$$-\frac{1}{f_{eq}} = 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{1}{f_m}$$

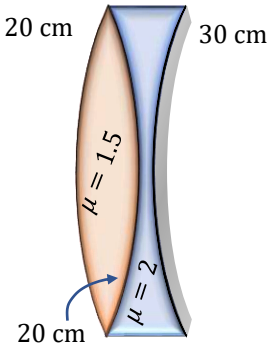
$$\boxed{\frac{1}{f_{eq}} = \frac{1}{f_m} - 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right)}$$




$f_1$, $f_2$ & $f_m$ should be put along with sign.

### Illustration 79:

Find out $P_{eq}$ & $f_{eq}$ in the following combination?




### Solution:

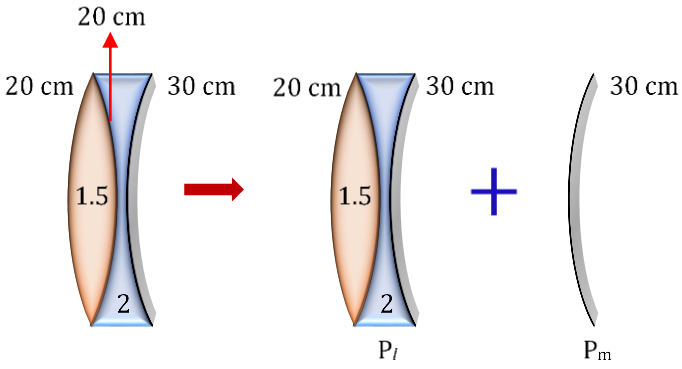
$$P_l = \left[\frac{1.5-1}{20} + \frac{2-1.5}{-20} + \frac{1-2}{30}\right]100$$

$$P_l = \left[\frac{1}{4} - \frac{1}{4} - \frac{1}{3}\right]10 = \frac{-10}{3}\text{D}$$

$$P_m = -\frac{1}{f} = -\frac{1}{R/2} = -\frac{100}{15} = -\frac{20}{3}\text{D}$$

$$P_{eq} = 2P_l + P_m = -\frac{20}{3} - \frac{20}{3} = -\frac{40}{3}\text{D}$$

$$f_{eq} = -\frac{1}{P_{eq}} = -\frac{100}{-40/3} = \frac{300}{40} = 7.5 \text{ cm}$$



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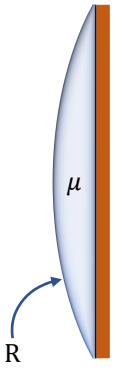
### Illustration 80:

Find out $P_{eq}$ & $f_{eq}$ in the following combination?

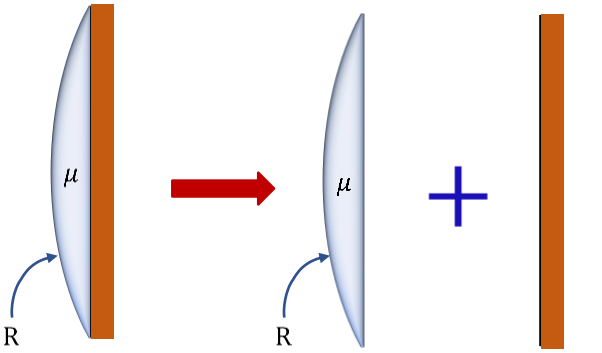



### Solution:

$$P_{eq} = 2P_l + P_m = \frac{2(\mu-1)}{R} + 0 = \frac{2(\mu-1)}{R}$$

$$\boxed{P_{eq} = \frac{2(\mu-1)}{R}} \Rightarrow f_{eq} = \frac{-R}{2(\mu-1)}$$

Positive (+ve) power indicates this combination will behave as a concave mirror of focal length $\frac{R}{2(\mu-1)}$.




$P_l = \frac{(\mu-1)}{R}$ $\quad P_m = 0$

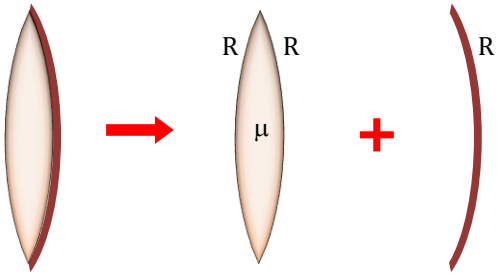
### Illustration 81:

Find out $P_{eq}$ & $f_{eq}$ in the following combination?

### Solution:

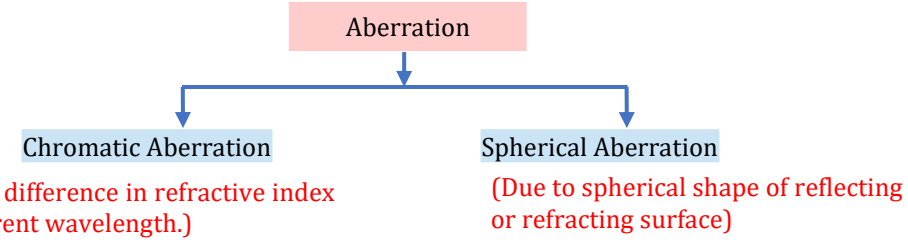
$$P_{eq} = 2P_l + P_m = \frac{2\times 2(\mu-1)}{R} + \frac{2}{R} = \frac{4\mu-2}{R}$$

$$f_{eq} = \frac{R}{4\mu-2}$$




## Aberration in Optical Elements

Aberration is a property of optical systems (such as lenses or mirror) that causes light to be spread out over some region of space rather than focused to a point.


Aberration
Chromatic Aberration
(Due to difference in refractive index of different wavelength.)
Spherical Aberration
(Due to spherical shape of reflecting or refracting surface)

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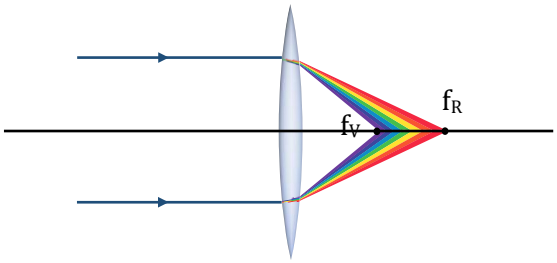
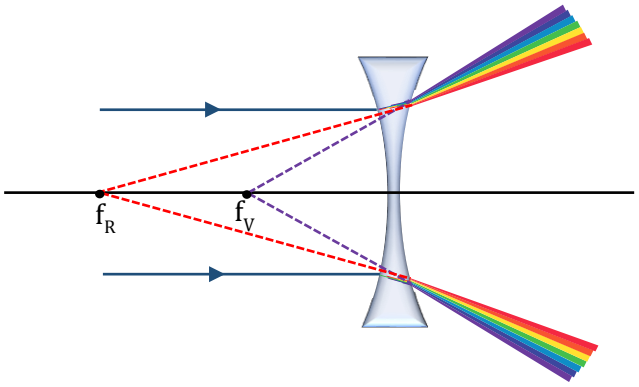
## Chromatic Aberration

The image of an object due to white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration which arises due to the fact that the focal length of a lens is different for different colours. For a single lens $\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ and as $\mu$ of lens is maximum for violet while minimum for red so, violet gets focussed nearest to the lens while red is farthest from it.




**Longitudinal or Axial Chromatic Aberration**

When an object O situated on the axis of a lens is illuminated by white light, then images of different colours are formed at different points along the axis. The formation of images of different colours at different positions is called 'axial' or longitudinal chromatic aberration. The axial distance between the red and the violet images $I_R - I_V$ is a measure of longitudinal aberration. When white light is incident on lens, image is obtained at different point on the axis because focal length of lens depends on wavelength.




$f \propto \lambda \Rightarrow f_R > f_V$.

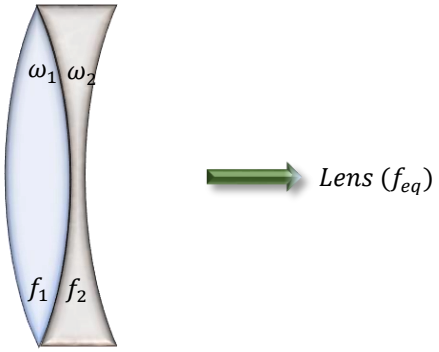
**$f_R - f_V = \omega f_y \Rightarrow$ Axial or longitudinal chromatic aberration**

If the object is at infinity, then the longitudinal chromatic aberration is equal to the difference in focal–lengths ($f_R - f_V$) for the red and the violet rays.

## Achromatism

If two or more lenses are combined together in such a way that this combination produces images due to all colours at the same point then this combination is known as achromatic combination of lenses. Focus for all colours are same.

To remove chromatic aberration, we use combination of converging and diverging lens.




**Condition for achromatism**

[when two lenses are in contact].

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_{eq}}$$

### Illustration 82:

The dispersive power of the materials of the two lenses are in the ratio 4:3. If the achromatic combination of these two lenses in contact is a convex lens of focal length 60 cm. Find out focal length of each lens.

### Solution:

Given, $\frac{\omega_1}{\omega_2}=\frac{4}{3}$ and $f_{eq}$ = 60 cm

For achromatic combination, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$

$$\frac{\omega_1}{\omega_2}=-\frac{f_1}{f_2}=\frac{4}{3} \quad \text{...(i)}$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_{eq}}$$

Put the value of $f_1$ from equation (i)

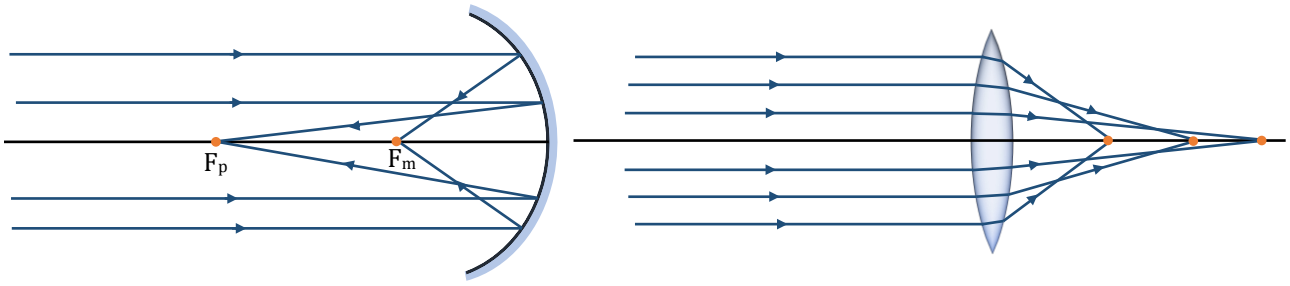
$$-\frac{3}{4f_2}+\frac{1}{f_2}=\frac{1}{60}\Rightarrow f_2=15 \text{ cm}$$

Put the value of $f_2$ from equation (i)

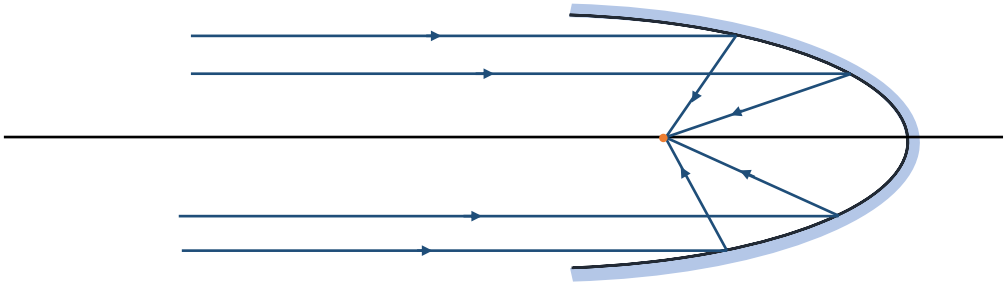
$$-\frac{f_1}{15}=\frac{4}{3}\Rightarrow f_1=-20 \text{ cm}$$

### Spherical Aberration

When all rays are incident on a spherical mirror or in lens in a direction parallel to the axis, the marginal rays come to focus at a point nearest to the pole called the marginal focus ($F_m$). The paraxial (or central) rays come to focus further away at '$F_p$'. Thus, the image of the distant object is not formed at one point but is spread along the axis between '$F_m$' and '$F_p$' this defect is called spherical aberration.



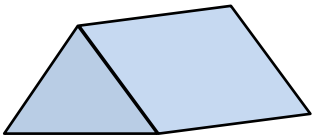

To remove spherical aberration in mirror we use parabolic reflectors.



## BEGINNER'S BOX-7

1. In the displacement method the distance between the object and the screen is 70 cm and the focal length of the lens is 16 cm, find the separation between the magnified and diminished image position of the lens.
2. An achromatic doublet of focal length 90 cm is to be made of two lens. The material of one of the lenses has 1.5 times the dispersive power of the other. The doublet is converging type. Find the focal length of each lens.
3. The dispersive power of material of a lens of focal length 20 cm is 0.08. Find the longitudinal chromatic aberration of the lens?
4. The dispersive powers of the materials of the two lenses are in the ratio of 4/3. If the focal length of this achromatic combination is 60 cm, find the focal length of the lenses.
5. What is the axial chromatic aberration in case of a lens which focusses violet ray 20.1 cm and red ray 20.3 cm away from it.

## Refraction Through Prism



An optical prism is a homogeneous solid transparent medium (such as glass) with flat surfaces that refract light. In prism light ray crosses two plane inclined surfaces. These surfaces are called the 'refracting surfaces' and the angle between them is called the 'refracting angle' or the 'angle of prism'.

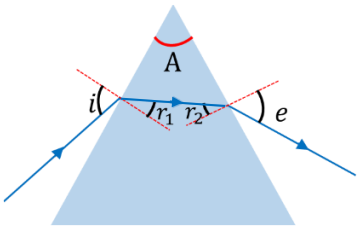
i : angle of incidence.

e : angle of emergence.

$r_1$ & $r_2$ are refracting angle inside the prism.

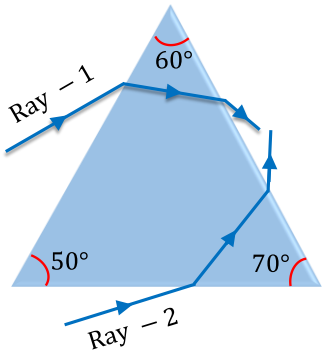
A : apex angle of prism OR prism angle

OR Refracting angle of prism.




Front view

### Calculation of Prism Angle




For Ray-1

A = 60°

For Ray-2

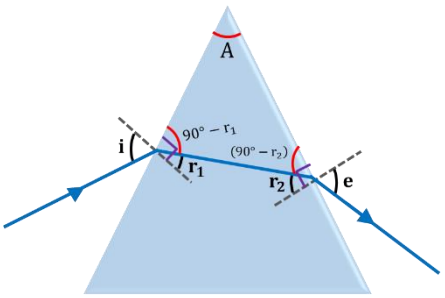
A = 70°

### Important results regarding Prism

$A + (90°-r_1) + (90°-r_2) = 180°$

$$\boxed{A = r_1 + r_2}$$

When required, $r_1$ & $r_2$ to be found using Snell's Law.

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**Deviation produced by Prism**

$\delta_1 = i - r_1$ (clockwise)

$\delta_2 = e - r_2$ (clockwise)

In both refraction, light ray deviated in same sense, hence,

$\delta_{net} = \delta_1 + \delta_2$ (clockwise)

$\delta_{net} = (i - r_1) + (e - r_2)$

$\delta_{net} = (i + e) - (r_2 + r_2)$

$$\boxed{\delta_{net} = (i + e) - A}$$

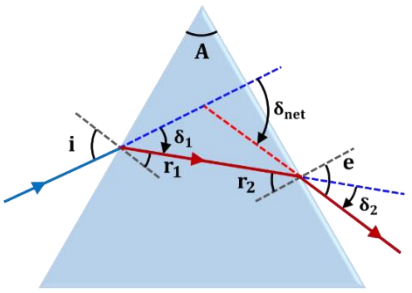



**Illustration 83:**

Find out deviation produce by prism in a light ray?

**Solution:**

Snell's law at first surface,

$1\sin 60° = \sqrt{3}\sin r_1$

$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1 \Rightarrow r_1 = 30°$

$A = r_1 + r_2$

$60° = 30° + r_2 \Rightarrow r_2 = 30°$

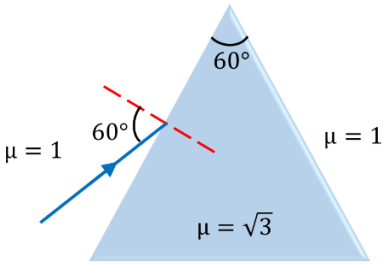
Snell's law at second surface

$\sqrt{3}\sin r_2 = 1\sin e.$

$\sqrt{3}\sin 30° = \sin e \Rightarrow e = 60°$

$\delta_{net} = i + e - A$

$\delta_{net} = 60° + 60° - 60° = 60°$



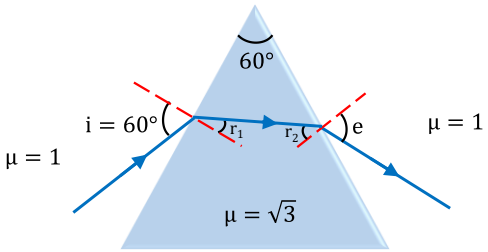




**Illustration 84:**

Find out deviation produce by prism in a light ray?

**Solution:**

Snell's law at first surface,

$1\sin 60° = \sqrt{3}\sin r_1$

$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1 \Rightarrow r_1 = 30°$

$A = r_1 + r_2$

$75° = 30° + r_2 \Rightarrow r_2 = 45°$

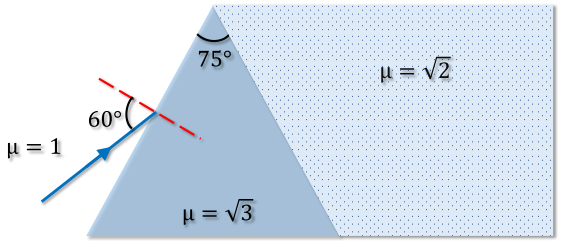
Snell's law at second surface,

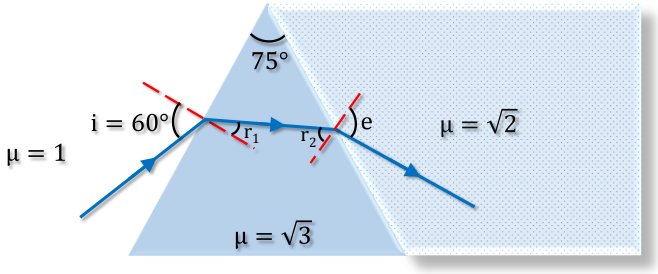
$\sqrt{3}\sin r_2 = \sqrt{2}\sin e.$

$\sqrt{3}\sin 45° = \sqrt{2}\sin e \Rightarrow e = 60°$

$\delta_{net} = i + e - A$

$\delta_{net} = 60° + 60° - 75° = 45°$

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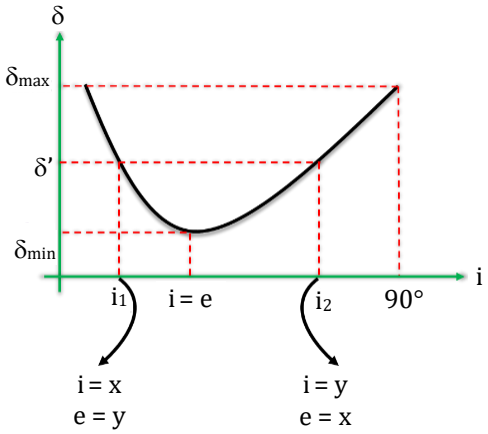
**δ v/s I curve for prism**




(1) The deviation produced by a prism depends on angle of incidence i, angle of prism A, refractive index of material μ.

(2) δ is first decreasing and then increasing for i < e and i > e respectively.

(3) For one δ (except $\delta_{min}$) there are two values of angle of incidence. If I and e are interchanged then we get the same value of δ because of reversibility principle of light.

(4) There is one and only one angle of incidence for which the angle of deviation is minimum (when i = e).

(5) Before i = e, δ decreases more rapidly than it increases with i after i = e. Right hand side part of the graph is more tilted then the left hand side.

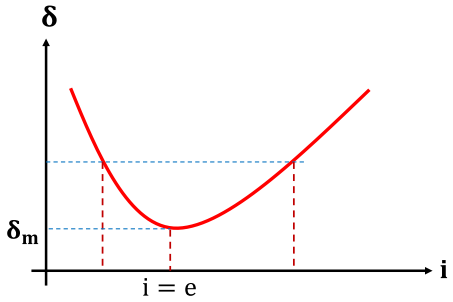
## Minimum Deviation in Prism

From graph condition of minimum deviation is

$$\boxed{i = e}$$

Then,

$$\boxed{r_1 = r_2 = \frac{A}{2}}$$




## Relation between Minimum deviation and Refractive index of prism

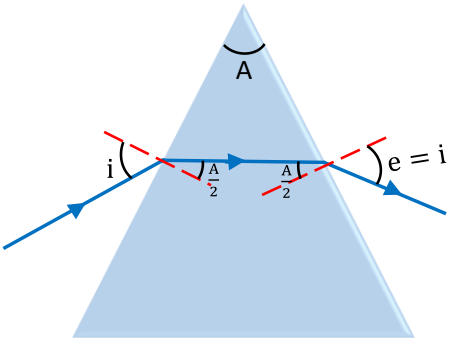
From $\delta = i + e - A$

$$\boxed{\delta_m = 2i - A} \Rightarrow i = \frac{\delta_m + A}{2}$$

Now Snell's law at first surface, $\mu_s \sin i = \mu_p \sin\left(\frac{A}{2}\right)$

$$\mu_s \sin\left(\frac{\delta_m + A}{2}\right) = \mu_p \sin\left(\frac{A}{2}\right)$$

$$\boxed{\frac{\mu_p}{\mu_s} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$




**Note:** For an isosceles / equilateral prism, light goes parallel to the base inside the prism.

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**Illustration 85:**

If refractive index of prism is $\sqrt{2}$ and refracting angle is 60° then find out minimum deviation produce by the prism.

**Solution:**

$$\frac{\mu_p}{\mu_s}=\frac{\sin\left(\frac{\delta_m+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}\Rightarrow\frac{\sqrt{2}}{1}=\frac{\sin\left(\frac{\delta_m+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\sqrt{2}=\frac{\sin\left(\frac{\delta_m+60°}{2}\right)}{\sin 30°}\Rightarrow\frac{1}{\sqrt{2}}=\sin\left(\frac{\delta_m+60°}{2}\right)\Rightarrow\frac{\delta_m+60°}{2}=45°\Rightarrow\delta_m=30°$$

**Illustration 86:**

An equilateral prism provides least deviation 46°in air. Find out the refractive index of unknown liquid in which same prism gives least deviation 30°

**Solution:**

Using formula, $\frac{\mu_p}{\mu_s}=\frac{\sin\left(\frac{\delta_m+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

For air, $\frac{\mu_p}{1}=\frac{\sin\left(\frac{46°+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$ ...(i)

For liquid, $\frac{\mu_p}{\mu_\ell}=\frac{\sin\left(\frac{30°+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$ ...(ii)

By equation (i) and (ii)

$$\mu_\ell=\frac{\sin 53°}{\sin 45°}=\frac{4\times\sqrt{2}}{5\times1}=\frac{4\sqrt{2}}{5}$$

**Illustration 87:**

A ray of light passing through a prism having refractive index $\sqrt{2}$ suffers minimum deviation. It is found that angle of incidence is double the angle of refraction within the prism. Find out angle of prism.

**Solution:**

For minimum deviation, $r_1 = r_2 = \frac{A}{2}$ and $i = 2r_1 \Rightarrow i = A$

Snell's law at first surface, $1 \times \sin i = \mu_p \sin r_1$

$$\sin A = \sqrt{2}\sin\left(\frac{A}{2}\right)$$

$$2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right) = \sqrt{2}\sin\left(\frac{A}{2}\right)$$

$$\cos\left(\frac{A}{2}\right)=\frac{1}{\sqrt{2}}\Rightarrow\frac{A}{2}=45°\Rightarrow A=90°$$

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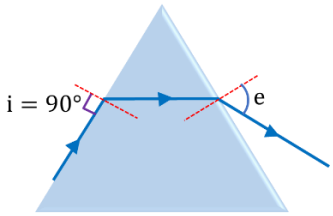
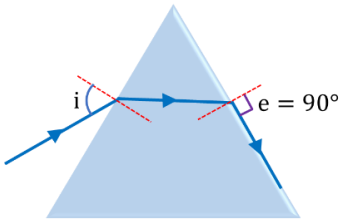
**Maximum Deviation**

For $\delta_{max}$ either i=90° or e=90°

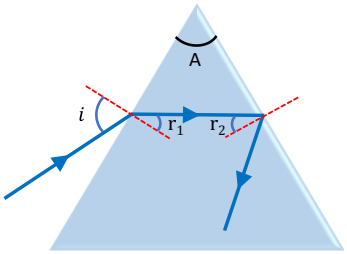



'OR'




Then use $\delta_{net} = (i + e) - A$

**Condition of No emergence**




$(r_2)_{min} > \theta_c$

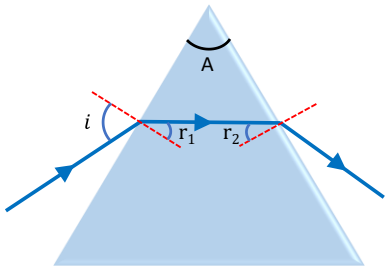
$A - r_{1\,max} > \theta_c$

When $i_{max} = 90°$ then $r_{1\,max} = \theta_c$

So, $A - \theta_c > \theta_c$

$\boxed{A > 2\theta_C}$

**Condition of Always emergence**




$(r_2)_{max} < \theta_c$

$A - r_{1\,min} < \theta_c$

When $i_{min} = 0°$ then $r_{1\,min} = 0°$

So, $\boxed{A < \theta_C}$

## Conclusion

1. if $A \le \theta_c$ ; then light ray always crosses the prism for any value of i.
2. if $A > 2\theta_c$ ; then light ray never crosses the prism for any value of i.
3. if $\theta_c < A \le 2\theta_c$ ; then light ray may crosses the prism or not depends on angle of incidence.

## Illustration 88:

Find out the maximum value of refractive index of a prism which permits the transmission of light through it when the refracting angle of the prism is 90°.

## Solution:

We can transmit the light through the prism when.

$$A \le \theta_c \Rightarrow \frac{A}{2} \le \theta_C$$

$$\sin\left(\frac{A}{2}\right) \le \sin\theta_C \Rightarrow \sin\left(\frac{90°}{2}\right) \le \frac{1}{\mu}$$

$$\mu \le \sqrt{2} \Rightarrow \mu_{max} = \sqrt{2}$$

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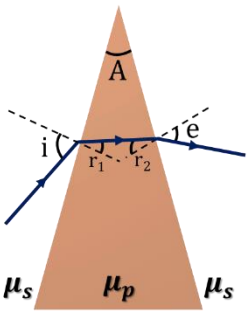
## Thin Prism

For a thin prism, prism angle (A) is very small.

In general A < 10°

In prism,



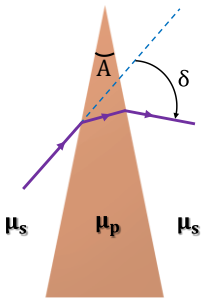

$$\frac{\mu_p}{\mu_s}=\frac{\sin\left(\frac{\delta_m+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If prism is thin then A is very small & then we assume all angles are very small

So, all apply $\sin\theta \approx \theta$ then,

$$\frac{\mu_p}{\mu_s}=\frac{\left(\frac{\delta_m+A}{2}\right)}{\left(\frac{A}{2}\right)} \Rightarrow \frac{\mu_p}{\mu_s}\times\frac{A}{2}=\frac{\delta_m+A}{2}$$

$$\delta_m=\left(\frac{\mu_p}{\mu_s}-1\right)A$$




General deviation or minimum deviation of thin prism is,

$$\boxed{\delta=\left[\frac{\mu_p}{\mu_s}-1\right]A}$$

The deviation for a small angled prism is independent of the angle of incidence.

### Illustration 89:

A thin prism of 5° angle gives a deviation of 3·2°. Find the refractive index of the material.

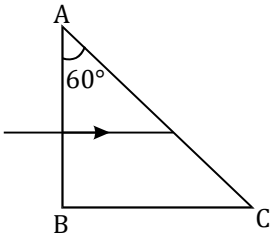
### Solution:

Angle of deviation $\delta = A(\mu - 1) \Rightarrow 3{\cdot}2° = 5°\,(\mu - 1) \Rightarrow \mu = 1{\cdot}64$

## BEGINNER'S BOX-8

**1.** Angle of incidence is 45° in the condition of minimum deviation for a prism of refracting angle 60°. Find the angle of deviation.

**2.** A light ray is incident normally on the surface AB of a prism of refracting angle 60°. If the light ray does not emerge from AC, then find the refractive index of the prism.




**3.** Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is $\frac{\pi}{3}$ radian.

**4.** A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation. It is found that angle of incidence is double the angle of refraction within the prism. Find angle of the prism.

**5.** The angle of minimum deviation measured with a prism is 30° and the angle of prism is 60°. Find the refractive index of the material of the prism.

**6.** A ray incident at 15° on a refracting surface of a prism of angle 30° suffers a deviation of 55°. Find the angle of emergence.

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## Dispersion of Light

Cauchy's Equation,

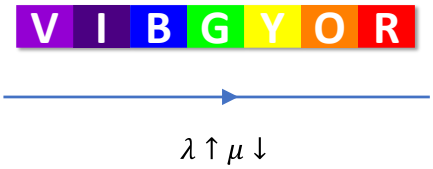
$$\mu(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots\ldots$$

Where A, B, C are positive constant.

$\mu$ depends on wavelength, $\lambda \uparrow \mu \downarrow$

- For vacuum (air) $\mu$ is same for all wavelength but in any other medium $\mu$ is different for different wavelength.

**For white light**




$\lambda_V < \lambda_R$ ; $\mu_V > \mu_R$

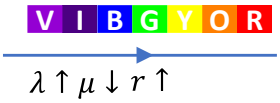
### Dispersion

When white light enters into a medium from air then the angular splitting of a ray of white light into a number of component and colour spreading in different directions is called Dispersion of Light.

$1 \times \sin i = \mu \times \sin r$

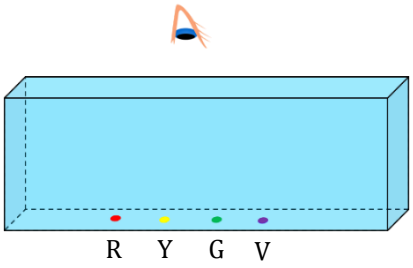
$$\sin r = \frac{\sin i}{\mu}$$

i is same for all colours.




### Illustration 90:

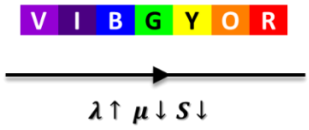
A plane glass slab is kept over various coloured letters; the letter which appears least raised is:

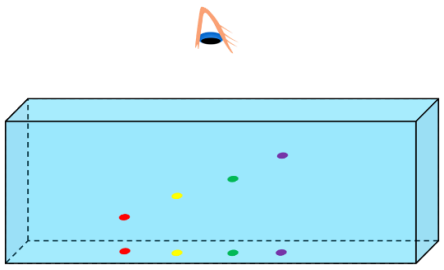



(1) Yellow (2) violet (3) green (4) red

### Solution:

Shift $S = t\left(1 - \frac{1}{\mu}\right)$



$S_V > S_R$ ; Red colour appears least raised.

So option (4) is correct.

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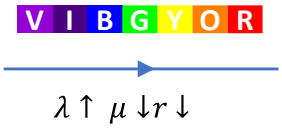
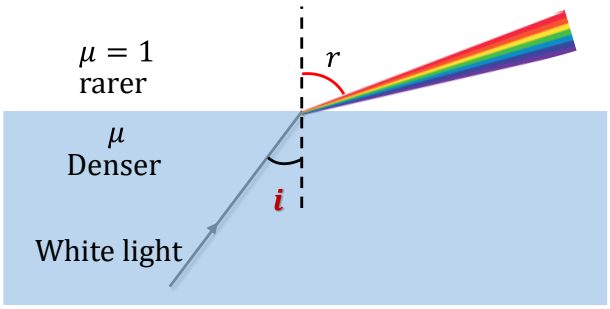
### Dispersion when white light goes from denser to rarer

$\mu \times \sin i = 1 \times \sin r$

i is same for all colours.




### Illustration 91:

White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains

(1) Yellow, orange, red

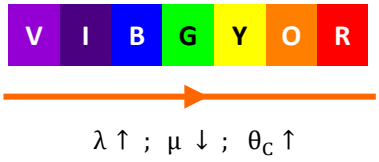
(2) Violet, indigo, blue

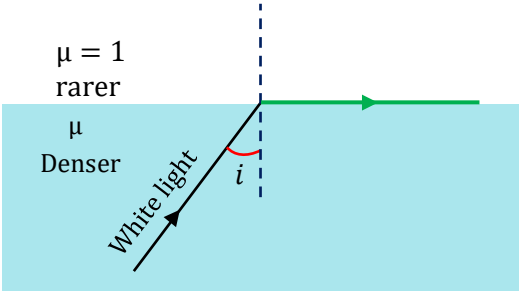
(3) All colours

(4) All colours except green

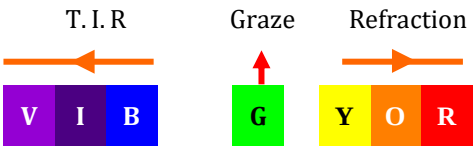
### Solution:

$\sin\theta_c = \frac{1}{\mu}$



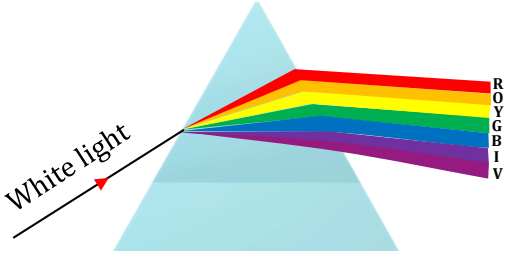

So




So, option (1) is correct.

### Dispersion through prism

When white light is incident on a prism then it is split into seven colours. This phenomenon is known as dispersion. Prism introduces different refractive indices with different wavelengths.

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## White light on thin prism

For thin prism, $\boxed{\delta = (\mu - 1)A}$

For violet colour, $\delta_v = (\mu_v - 1)A$

For red colour, $\delta_R = (\mu_R - 1)A$

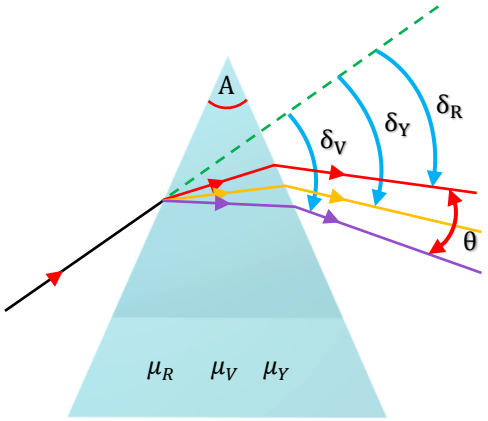
**Angular dispersion**

It is the difference between the angles of deviation for violet colour and red colour.

Angular dispersion, $\theta = \delta_V - \delta_R = (\mu_V - 1)A - (\mu_R - 1)A$

$\boxed{\theta = \delta_V - \delta_R}$

$\boxed{\theta = (\mu_v - \mu_R)A}$




It depends on prism material and on the angle of prism.

## Dispersive power (ω)

It is the ratio of angular dispersion to mean deviation.

$$\omega = \frac{\theta}{\delta} \Rightarrow \boxed{\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}}$$

Dispersive power depends only on the material of the prism.

Refractive index of mean colour, $\mu_Y = \frac{\mu_V + \mu_R}{2}$.

### Illustration 92:

White light is passed through a prism of angle 6°. If the refractive indices for red and violet colours are 1.6 and 1.7 respectively, calculate the angle of dispersion between them.

### Solution:

$\theta = [\mu_V - \mu_R]A = [1.7 - 1.6]\,6° = 0.6°$

### Illustration 93:

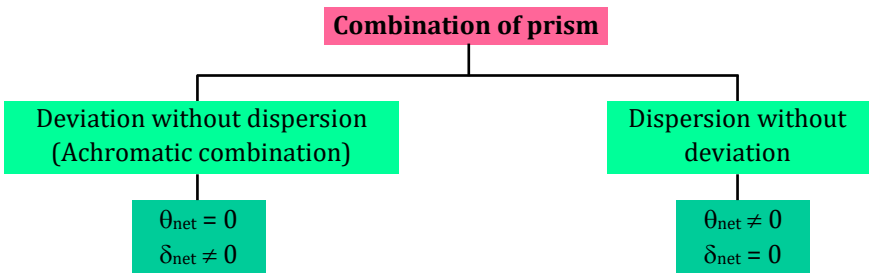
Prism angle of a prism is 6°. Their refractive index for red and violet colours is 1.6 and 1.8 respectively. Then find the dispersive power.

### Solution:

Dispersive power of prism, $\omega = \left(\frac{\mu_v - \mu_r}{\mu_y - 1}\right)$ but $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.8 + 1.6}{2} = 1.7$

Therefore, $\omega = \left[\frac{1.8 - 1.6}{1.7 - 1}\right] = \left[\frac{0.2}{0.7}\right] = 0.117$

## Combination of prism



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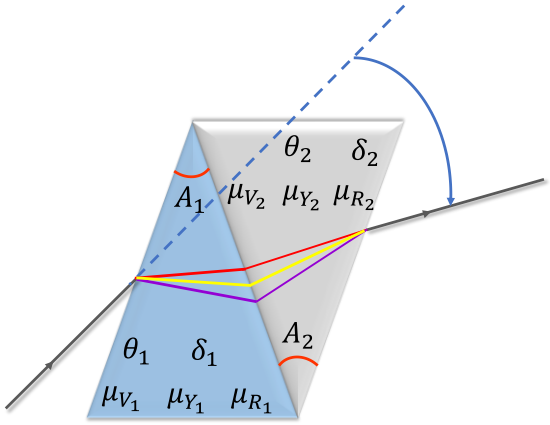
## Deviation without dispersion (Achromatic Combination)

Two or more thin prisms are combined in such a way that deviation occurs i.e. emergent light ray makes certain angle with incident light ray but dispersion does not occur i.e., white light is not split into different colours.




Without dispersion means

$\theta_{net} = 0$

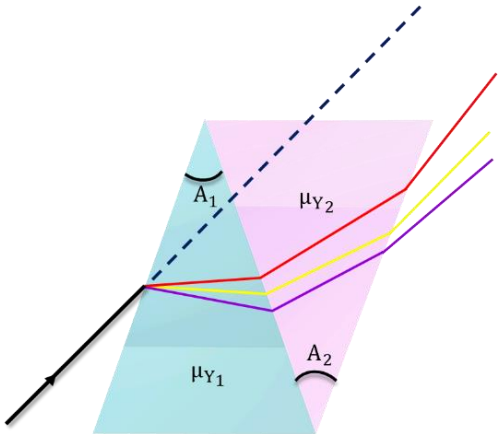
$\theta_1 - \theta_2 = 0$

$(\mu_{V_1} - \mu_{R_1})A_1 - (\mu_{V_2} - \mu_{R_2})A_2 = 0$

$$\boxed{\frac{A_1}{A_2} = \frac{\mu_{v_2} - \mu_{R_2}}{\mu_{v_1} - \mu_{R_1}}}$$

## Dispersion without deviation

Two or more thin prisms combine in such a way that dispersion occurs i.e., white light is splitted into different colours but mean deviation does not occur i.e., emergent light ray remains parallel to incident light ray.




Without deviation means

$\delta_{net} = 0$

$\delta_1 - \delta_2 = 0$

$(\mu_{Y_1} - 1)A_1 - (\mu_{Y_2} - 1)A_2 = 0$

$$\boxed{\frac{A_1}{A_2} = \frac{\mu_{Y_2} - 1}{\mu_{Y_1} - 1}}$$

### Illustration 94:

Two prism are combined to form an achromatic combination

| Prism -1 | Prism -2 |
|---|---|
| $A = 4°$ | $A = ?$ |
| $\mu_R$ =1.35 | $\mu_R$ =1.7 |
| $\mu_Y$ =1.40 | $\mu_Y$ =1.8 |
| $\mu_v$ =1.42 | $\mu_v$ =1.9 |

### Solution:

For a achromatic combination,

$\theta_{net} = 0 \Rightarrow \theta_1 = \theta_2$

$(\mu_{V_1} - \mu_{R_1})A_1 = (\mu_{V_2} - \mu_{R_2})A_2$

$[1.42 - 1.35]\ 4° = [1.9 - 1.7]\ A \Rightarrow A = 1.4°$

$\delta_{net} = \delta_1 - \delta_2 = [1.40 - 1]\ 4° - [1.8 - 1]\ 1.4° = 0.48°$

### Illustration 95:

A light ray is incident on a thin prism of angle 3° having refractive index 1.4, it is combined with another prism of refractive index 1.6 to produce dispersion without deviation. Find the prism angle of another prism.

### Solution:

For dispersion without deviation, $\delta_{net} = 0 \Rightarrow \delta_1 = \delta_2$

$\left[\mu_{y_1} - 1\right]A_1 = \left[\mu_{y_2} - 1\right]A_2 \Rightarrow [1.4 - 1]\ 3° = [1.6 - 1]\ A_2 \Rightarrow A_2 = 2°$

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**Note:**

- Dispersive power like refractive index has no units and dimensions and depends on the material of the prism and is always positive.
- As for a given prism dispersive power is constant. Deviation is maximum for violet and minimum for red.
- As for a given prism $\theta \propto \delta$ a single prism produces both deviation and dispersion simultaneously, i.e., a single prism cannot give deviation without dispersion or dispersion without deviation.

## BEGINNER'S BOX-9

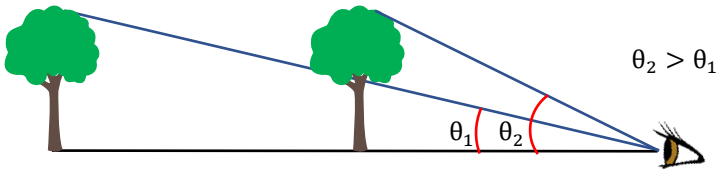
**1.** White light is passed through a prism of angle 5°. If the refractive index for red and blue colours are 1·641 and 1·659 respectively, then find the angle of dispersion between them.

**2.** White light is passed through a prism of angle 10°. If the refractive index for red and violet colours are 1.641 and 1.659 respectively, then find the-
(a) angles of deviation for violet and red colours.
(b) angular dispersion
(c) dispersive power

**3.** For a certain material the refractive indices for red, violet and yellow colour lights are 1.52, 1.64 and 1.60 respectively. Find the dispersive power of the material.

## Optical Instruments (Simple microscope, Compound microscope, Telescope)

**Visual angle:** Visual angle is the angle, a viewed object or image subtends at the eye. It is also called the object's angular size.




Nearer the object larger the visual angle then object appears big in size.

**Note:** Near point (N.P.) of normal eye D = 25cm
Far point (F.P.) of normal eye = ∞

## Angular Magnification 'OR' Magnifying power

$$\text{M.P.} = \frac{\text{visual angle of image in the presence of instrument}(\beta)}{\text{maximum visual angle of an object in the absence of instrument}(\alpha)}$$

## Simple Microscope

(1) A single converging lens (convex lens) of small focal length.

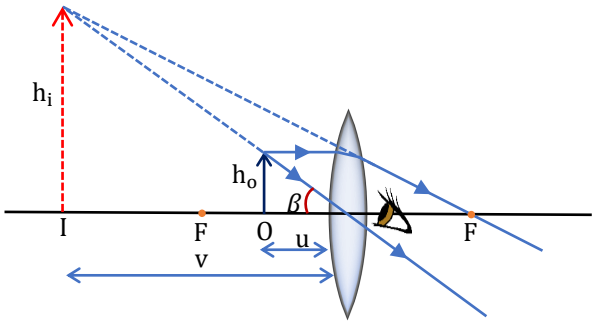
(2) When the object is placed between the focus and the optical centre a virtual, magnified and erect image is formed.

(3) Also known as magnifying glass.

$$\tan\beta = \frac{h_o}{u} = \frac{h_i}{v}$$

If angle is small then,

$$\beta = \frac{h_o}{u} = \frac{h_i}{v}$$

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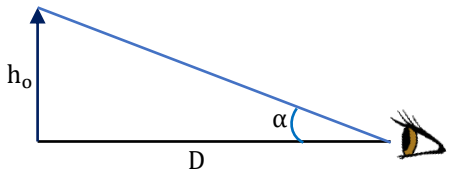
## Calculation of magnifying power

$$\tan\alpha \approx \alpha = \frac{h_o}{D}$$

$$M.P. = \frac{\beta}{\alpha} = \frac{h_o / u}{h_o / D}$$

$$\boxed{M.P. = \frac{D}{|u|}}$$




**Case (1):** When image formed at a distance of least distance of distinct vision from the lens.

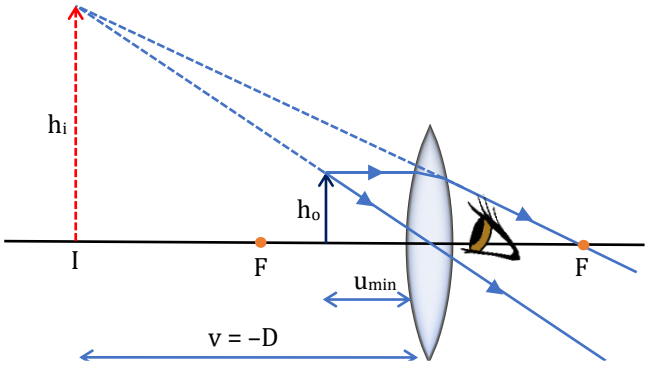
$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{D} + \frac{1}{f}$$

[v = –D and u = –ve]

Multiplying both the sides by D

$$\frac{D}{u} = 1 + \frac{D}{f} \Rightarrow MP = \frac{D}{u} = 1 + \frac{D}{f}$$

$$\boxed{(M.P.)_{max} = \frac{D}{u_{min}} = 1 + \frac{D}{f}}$$



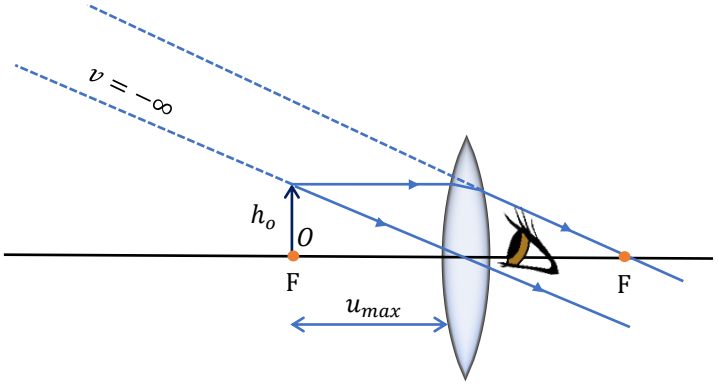

**Case (2):** When image formed at infinite distance from the lens.

From lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-\infty} - \frac{1}{-u} = \frac{1}{f} \Rightarrow u = f.$$

So $MP = \frac{D}{u} = \frac{D}{f}$

$$\boxed{(M.P.)_{min} = \frac{D}{u_{max}} = \frac{D}{f}}$$




### Illustration 96:

Focal length of a convex lens is 2.5 cm. Find its maximum and minimum magnifying power when used as a simple microscope.

### Solution:

$$(M.P.)_{max} = 1 + \frac{D}{f} = 1 + \frac{25}{2.5} = 11$$

$$\Rightarrow (M.P.)_{min} = \frac{D}{f} = \frac{25}{2.5} = 10$$

### Illustration 97:

A man with normal vision, reads a book with small print using a magnifying glass of focal length 5 cm. What is the closest and farthest distance at which he should hold the lens from the print so that he can read easily?

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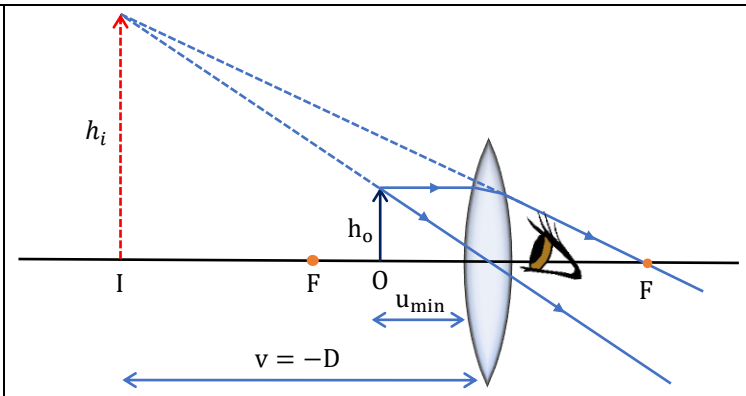
**Solution:**

$$\frac{D}{u_{min}} = 1 + \frac{D}{f}$$

$$\frac{25}{u_{min}} = 1 + \frac{25}{5}$$

$$u_{min} = \frac{25}{6}\ cm$$




$$\frac{D}{u_{max}} = \frac{D}{f}$$

$$u_{max} = f$$

$$u_{max} = 5\ cm$$

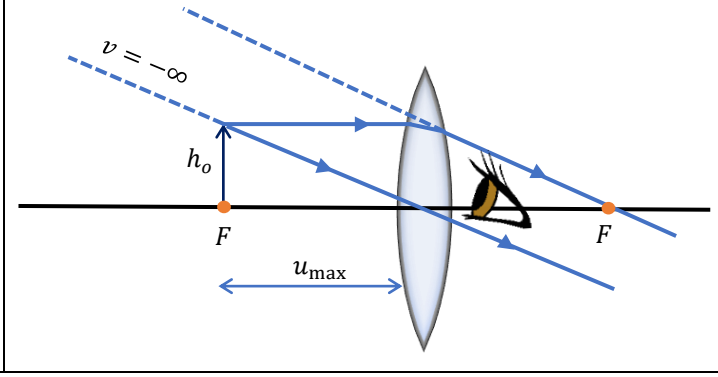



**Illustration 98:**

A 10 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification for a normal eye (near point = 25 cm)?

**Solution:**

**1st method**

$$P = \frac{1}{f} = 10$$

f = 0.1 m = 10 cm

$$(M.P)_{max} = 1 + \frac{D}{f} = 1 + \frac{25}{10}$$

$$= 3.5$$

$$M.P = \frac{D}{|u|} \Rightarrow 3.5 = \frac{25}{u} \Rightarrow u = \frac{50}{7}\ cm$$

**2nd method**

We get maximum M.P where using image formed at least distance of district vision

v = –25 cm

$$-\frac{1}{25} - \frac{1}{u} = \frac{1}{10} \Rightarrow \quad u = \frac{-50}{7}\ cm$$

**Compound microscope**

A compound microscope is a laboratory instrument with high magnification power, which is consists of more than one lenses. Compound Microscopes are used for the study of structural details of a cell, tissue, or organ in sections. A compound microscope can magnify the image of a tiny object up to 1000.

**Ray diagram:** Compound microscope is used to get more magnified image as compared to a simple microscope. Object is placed in front of the objective lens and the image is seen through the eye piece. The aperture of objective lens is less as compared to eye piece because object is very

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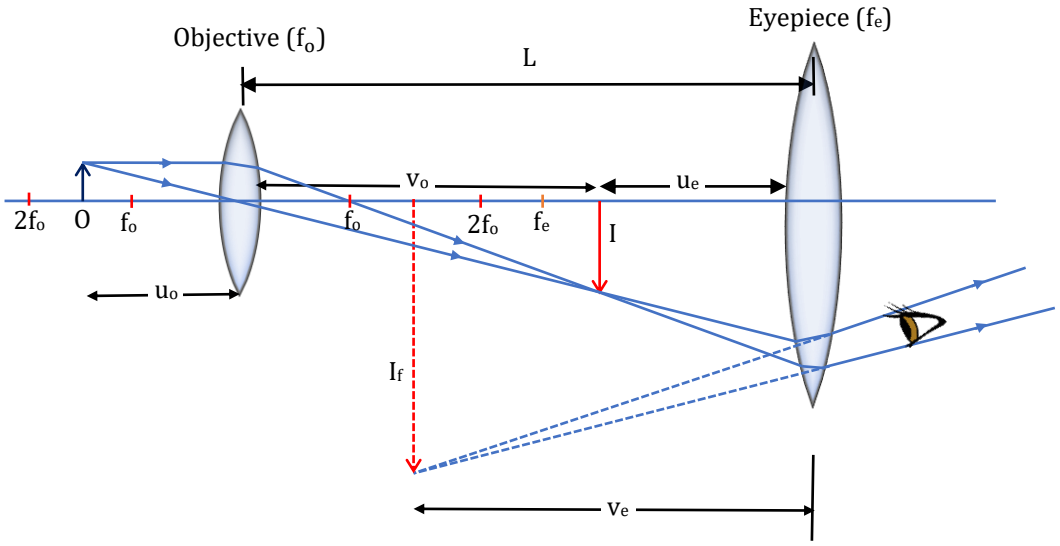
near so collection of more light is not required. Generally, object is placed between F – 2F due to this a real, inverted and magnified image is formed between 2F – ∞. It is known as intermediate image (I). The intermediate image acts as an object for the eye piece. Now the distance between both the lens are adjusted in such a way that intermediate image falls between the optical centre of eye piece and its focus. In this condition, the final image ($I_f$) is virtual, inverted and magnified.

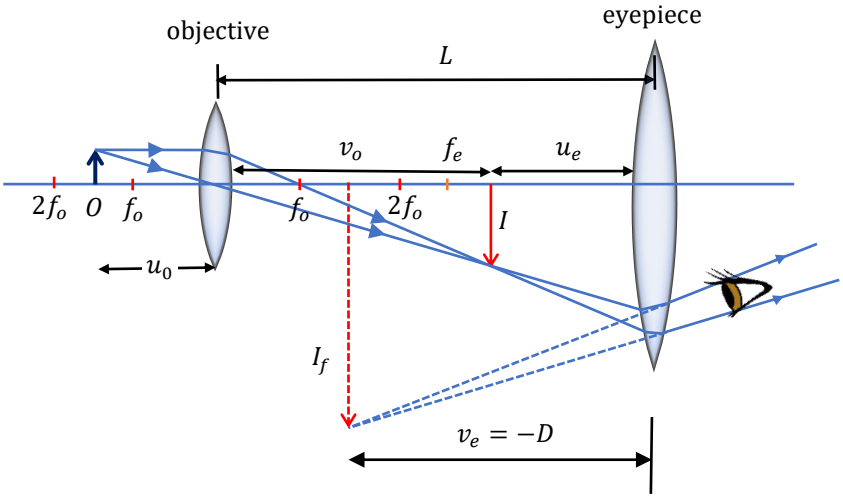



**Calculation of magnifying power:** Total magnifying power = Linear magnification of objective lens × angular magnification MP of eye lens

$$\mathbf{M.P. = m_o \times m_e}$$

$$\mathbf{M.P. = -\left|\frac{v_o}{u_o}\frac{D}{u_e}\right|}$$

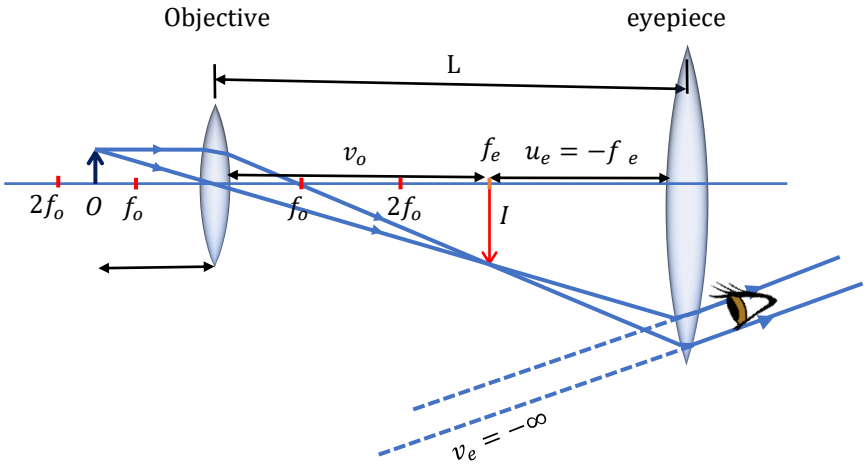
**Case (1):** When final image formed at least distance of distinct vision.




$$MP = \frac{v_0}{u_0}\left[1+\frac{D}{f_e}\right] = \frac{f_0}{(f_0+u_0)}\left[1+\frac{D}{f_e}\right] = \frac{f_0-v_0}{f_0}\left[1+\frac{D}{f_e}\right]$$

Distance between both the lenses, $L = v_0 + |u_e|$

**Case (2):** When final image formed at infinity.




$$MP = \frac{v_0}{u_0}\left[\frac{D}{f_e}\right] = \frac{f_0}{(f_0 + u_0)}\left[\frac{D}{f_e}\right] = \frac{(f_0 - v_0)}{f_0}\left[\frac{D}{f_e}\right]$$

Distance between both the lenses, $L = v_0 + f_e$

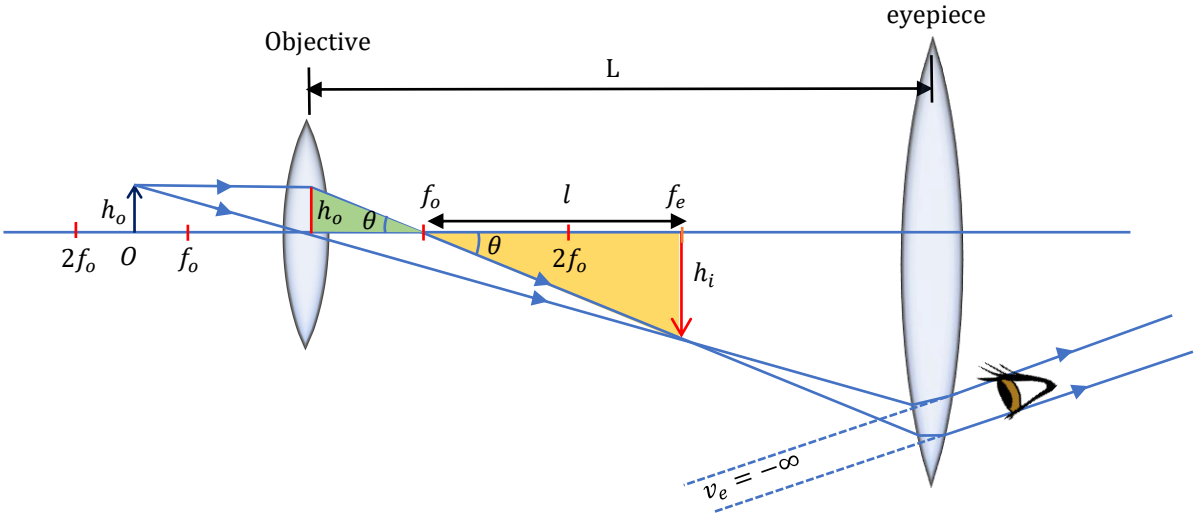
Sign convention for solving numericals, $u_0$ = –ve, $v_0$ = +ve, $f_0$ = +ve,

$u_e$ = –ve, $v_e$ = –ve, $f_e$ = +ve, $m_0$ = –ve, $m_e$ = +ve, MP = –ve

## Tube length

The distance *l*, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length $f_e$) is called the tube length of the compound microscope.

Magnifying power of compound microscope when final image formed at infinity.




$$\left.\begin{aligned}\tan\theta &= \frac{h_o}{f_o}\\ \tan\theta &= \frac{h_i}{l}\end{aligned}\right\} \Rightarrow \frac{h_o}{f_o} = \frac{h_i}{l} \Rightarrow \frac{h_i}{h_o} = \frac{l}{f_o}$$

So, M.P. of compound microscope when final image formed at ∞.

M.P. = $m_o \times m_e$

$$\text{M.P.} = -\frac{l}{f_o} \times \frac{D}{f_e}$$

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**Illustration 99:**

A compound microscope has a magnifying power 30X. The focal length of its eye-piece is 5 cm. Assuming the final image formed at the least distance of distinct vision (25cm). Calculate the magnification produced by objective.

**Solution:**

Given, M.P. = –30, $f_e$ = 5 cm

M.P. of eyepiece when final image formed at 25 cm away,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Now, M.P. = $m_o \times m_e$

$-30 = m_o \times 6 \Rightarrow m_o = -5$

**Illustration 100:**

An angular magnification of 30X is desired using an objective of focal length 1.25 cm and an eye piece of focal length 5 cm. What distance will you set between Objective and Eye piece of the compound microscope for final image at infinity?

**Solution:**

Given, M.P. = –30, $f_o$ = 1.25 cm, $f_e$ = 5 cm

If final image at $\infty$ then $m_e = \frac{D}{f_e} = \frac{25}{5} = 6$

M.P. = $m_o \times m_e$

$-30 = m_o \times 6 \Rightarrow m_o = -5$

$$m_o = \frac{f_o - V_o}{f_o} = -5 \Rightarrow \frac{1.25 - V_o}{1.25} = -5 \Rightarrow V_o = 7.5 \text{ cm}$$

Distance between objective and eyepiece is, $L = V_o + f_e = 7.5 + 5 = 12.5$ cm

**Illustration 101:**

In a compound microscope object is placed at a distance of 6 cm from objective of focal length 5 cm and focal length of eyepiece is 10 cm. Find out magnifying power of compound microscope when final image is formed at least distance of distinct vision?

**Solution:**

Given, $u_o$ = –6 cm, $f_o$ = 5 cm, $f_e$ = 10 cm

When final image formed at least distance of distinct vision then,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{10} = 3.5 \quad \& \quad m_o = \frac{f_o}{f_o + u_o} = \frac{5}{5-6} = -5$$

M.P. = $m_o \times m_e = -5 \times 3.5 = -17.5$

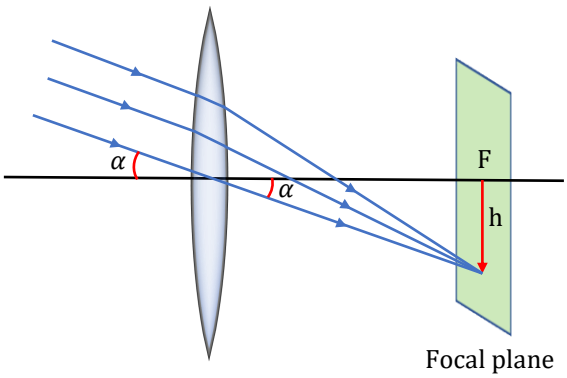
**Telescope**

**Focal Plane:** When light rays are parallel and paraxial but not parallel to principal axis can incident on the lens then the plane where they meet or appears to meet after refraction is known as focal plane.

$$\tan\alpha = \frac{h}{f}$$

If rays are paraxial then, $\tan\alpha \approx \alpha$

$$\alpha = \frac{h}{f} \Rightarrow \boxed{h = f\alpha}$$



Focal plane

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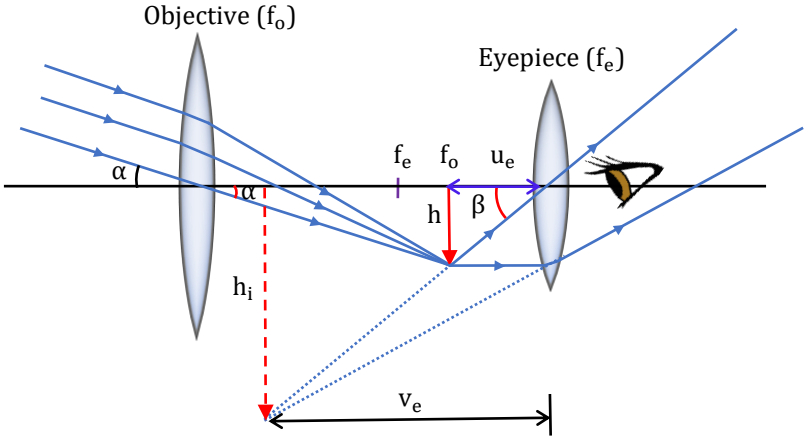
## Astronomical telescope

The telescope is used to provide angular magnification of distant objects (Fig.). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.




$h = f_o\alpha$

$$\alpha = \frac{h}{f_o}$$

$$\beta = -\frac{h}{u_e}$$

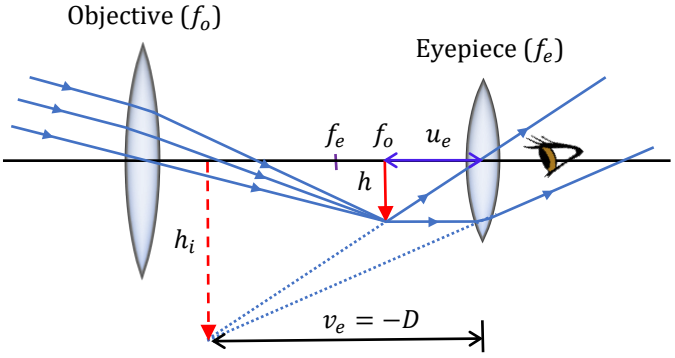
### Calculation of magnifying power

$$MP = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle for unaided eye } (\alpha)} \Rightarrow MP = \frac{\frac{h}{-u_e}}{\frac{h}{f_0}} = -\frac{f_0}{u_e}$$

$$\boxed{MP = -\frac{f_o}{u_e}}$$

**Case (1):** When final image formed at least distance of distinct vision, $v_e = -D$, $u_e$ = –ve

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$




So $\boxed{MP = -\frac{f_o}{u_e} = -\frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right]}$

Length of the tube is, $L = f_0 + |u_e|$

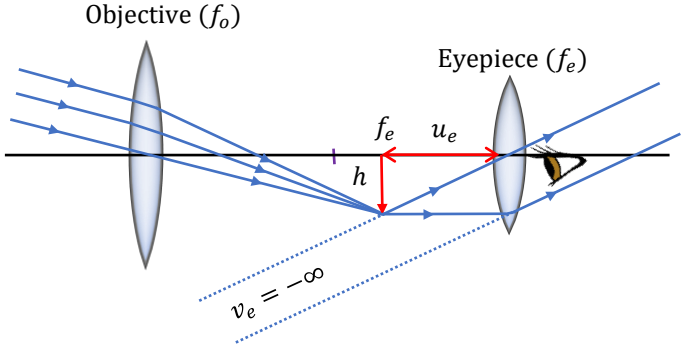
**Case (2):** When final image formed at infinity

$v_e = -\infty$, $u_e$ = –ve

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow u_e = f_e$$

So $\boxed{MP = -\frac{f_o}{f_e}}$

Length of the tube, $L = f_0 + f_e$



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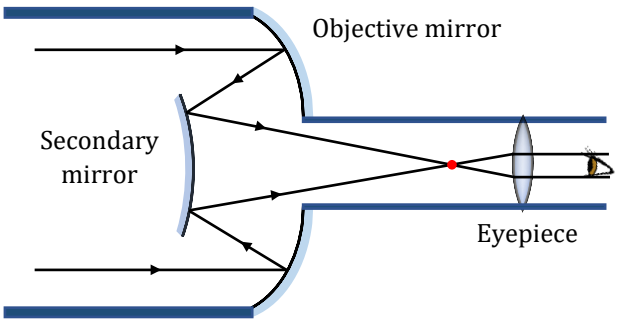
### Cassegrain telescope (Reflecting Telescope)




Big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions. For these reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called reflecting telescopes. There is no chromatic aberration in a mirror. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim. One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. Another solution to the problem is to deflect the light being focussed by another mirror.

One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, as shown in Figure. This is known as a Cassegrain telescope, after its inventor. It has the advantages of a large focal length in a short telescope.

### Illustration 102:

A small telescope has an objective of focal length 80 cm and an eyepiece of focal length 2 cm. If final image is formed at infinity.

(1) What is the magnifying power of the telescope?

(2) What is the separation between the objective and the eyepiece?

### Solution:

$$\text{M.P} = \frac{f_o}{f_e} = \frac{80}{2} = 40$$

$L = f_o + f_e = 80 + 2 = 82$ cm

### Illustration 103:

Magnifying power of an astronomical telescope is 5. If focal length of eye piece is 4 cm. Find focal length of objective lens & also find the distance between both the lenses when final image is formed at infinity.

### Solution:

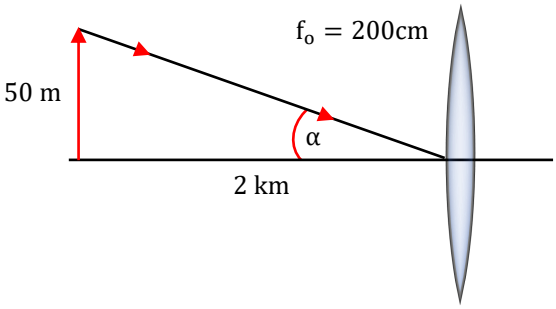
Given, M.P. = 5, $f_e = 4$ cm

$$\text{M.P.} = \frac{f_o}{f_e} \Rightarrow 5 = \frac{f_o}{4} \Rightarrow f_o = 20 \text{ cm}$$

$L = f_o + f_e = 20 + 4 = 24$ cm

### Illustration 104:

A telescope has an objective lens of focal length 200 cm. It is used to see a 50 m tall building at 2 km. What is height of the image formed by objective lens?




### Solution:

$$\tan\alpha = \frac{50}{2000} \quad (\alpha \text{ is small angle})$$

$$\alpha = \frac{1}{40}$$

$$h = f_o\alpha = 200 \times \frac{1}{40} = 5 \text{ cm}$$

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**Illustration 105:**

An astronomical telescope has objective and eyepiece of focal length 40 cm and 4 cm respectively. To view an object 200 cm away from the objective, the lens must be separated by a distance.

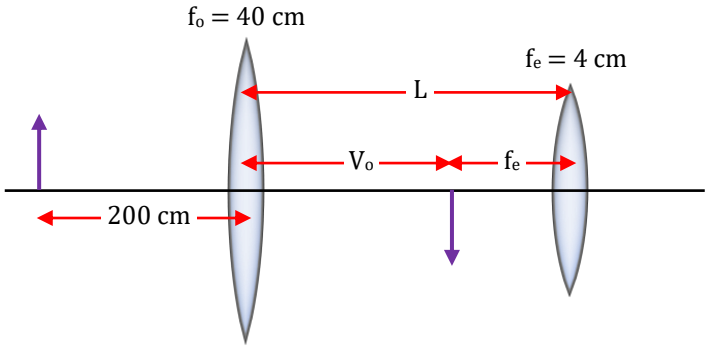
**Solution:**

Assume final image formed at $\infty$,

$$V_0 = \frac{u_o f_o}{u_o + f_o}$$

$$V_0 = \frac{-200 \times 40}{-200 + 40} = 50 \text{ cm}$$

$$L = V_o + f_e = 50 + 4 = 54 \text{ cm}$$



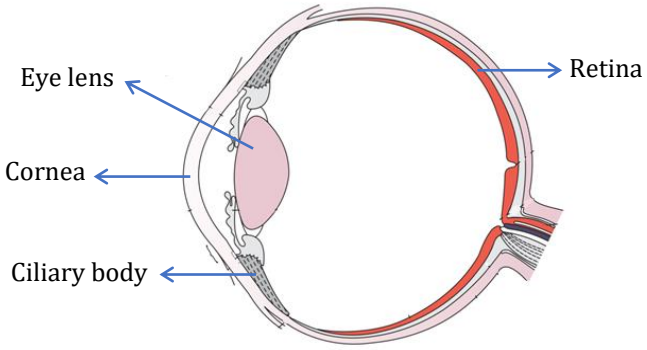

## BEGINNER'S BOX-10

1. Magnification of a compound microscope is 30. Focal length of eye piece is 5 cm and the image is formed at least distance of distinct vision. Find the magnification of objective.
2. The powers of the lenses of a telescope are 0.5 and 20 dioptres. If the final image is formed at the minimum distance of distinct vision (25 cm) then what will be length of the tube ?
3. The focal lengths of objective and eye piece of a Galilean telescope are respectively 30 cm and 5 cm. Calculate its magnifying power and length when used to view distant objects.
4. A telescope consisting of an objective of focal length 60 cm and an eyepiece of focal length 5 cm is focussed to a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then find the angular width of the image.
5. The focal lengths of the objective and the eye piece of an astronomical telescope are 60 cm and 5 cm respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) least distance of distinct vision (25 cm).

## Defects of Vision

### Human Eye

- Eye lens is converging in nature (convex in shape).
- A healthy eye always makes image on the Retina.
- So, for variable distance of object from eye, focal length of lens is variable.
- Ciliary muscles control focal length of lens by changing its radius of curvature.




### Far Point (F.P.)

- The maximum distance which a person can see without the help of spectacles.
- For healthy human eye, **Far point is taken at infinity.**
- Eye lens is **relaxed** while looking at Far Point.
- Radius of Curvature of eye lens is high.
- **Low Converging Power** is required.

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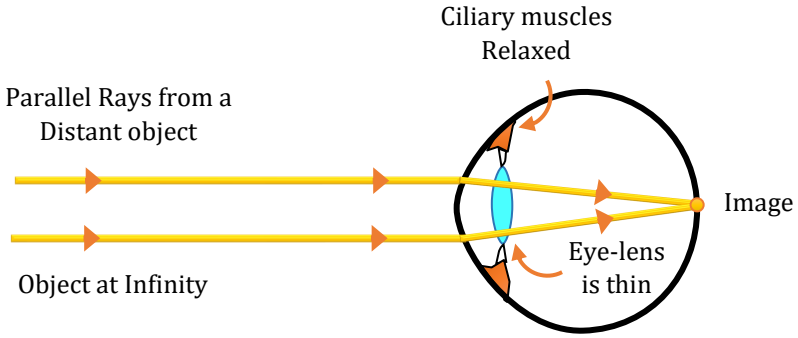
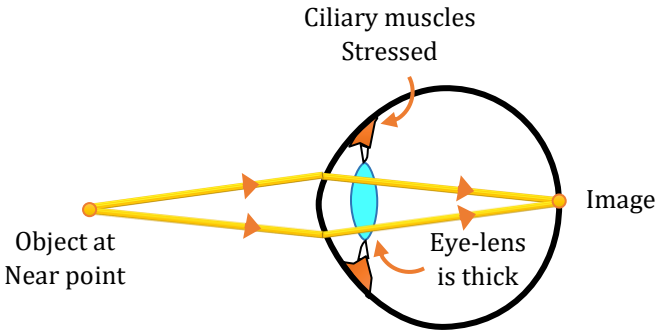
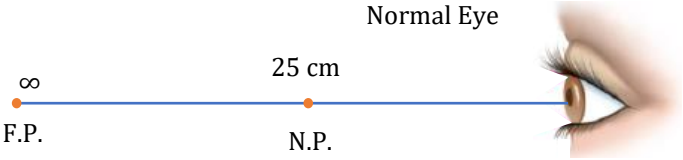



## Near Point (N.P.)

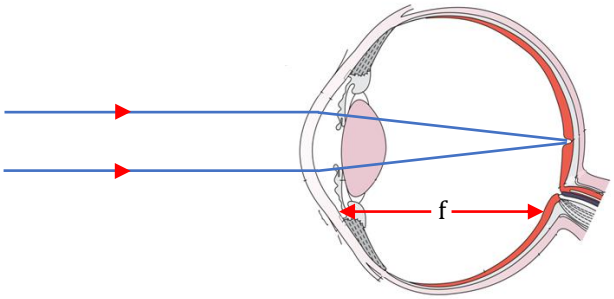
- In visual perception, the near point is the closest point at which an object can be placed and still form a focused image on the retina, within the eye's accommodation range.
- For healthy human eye, **near point is taken at 25 cm**. This is also known as least distance of distinct vision.
- Eye lens is **stressed** while looking at Near point.
- Radius of Curvature of eye lens is low.
- **High Converging Power** is required.




## Illustration 106:

For a normal eye, the cornea of eye provides a converging power of 40 D and the least converging power of the eye lens behind the cornea is 20 D. Using this information, the distance between the retina and the cornea eye lens can be estimated to be -




**Solution:**

Least converging power means object at ∞

$P_{total}$ = 40 + 20 = 60D

$f = \frac{1}{P}$

$f = \frac{100}{60}$ = 1.67 cm

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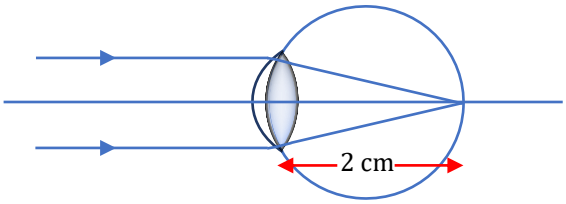
**Illustration 107:**

A normal eye has retina 2 cm behind the eye-lens. What is the power of the eye-lens when the eye is (a) fully relaxed, (b) most strained?

**Solution:**

(a) fully relaxed: When object at ∞ or light rays are parallel to each other.

$$P = \frac{1}{f} = \frac{100}{2} = 50D$$

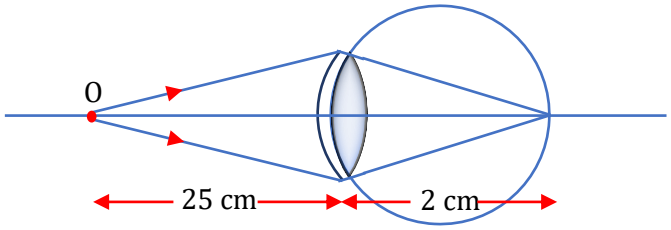



(b) most strained: When object at 25 cm away from eye lens them.

u = –25 cm, V = 2 cm

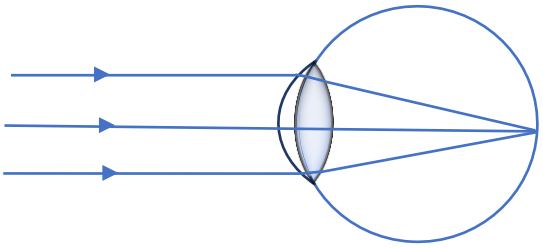
$$\frac{1}{V} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2} + \frac{1}{25} = \frac{1}{f}$$

$$f = \frac{50}{27} \text{ cm} \Rightarrow P = \frac{100}{50} \times 27 = 54D$$

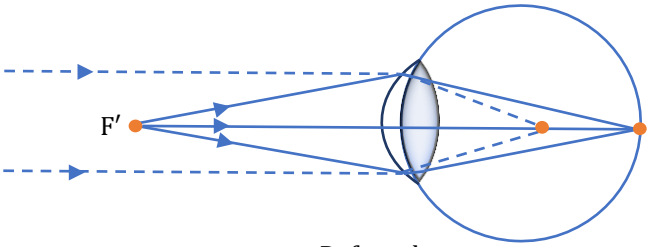



**Myopia (Near-sightedness)**

- In this defect nearby objects are clearly visible but distant object are not clearly visible.
- Far point of eyes is closer than infinity.



Normal eye
(far point at ∞)

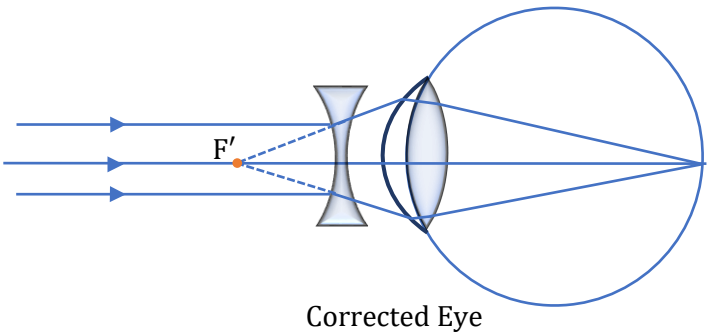



Defected eye
(far point at F′)

The most common symptom of myopia is blurred vision for distant objects.

**Correction of defect**

To remove this defect **diverging lens (concave lens)** is used.




Corrected Eye

**To solve the problems**

u = –∞; v = –F′

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad f = -F'$$

$$\frac{1}{-F'} - \frac{1}{-\infty} = \frac{1}{f} \qquad \boxed{P = \frac{1}{f} = -\frac{1}{F'}}$$

(where F' is the new far point of eye)

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### Illustration 108:

A person cannot see clearly an object kept at a distance beyond of 100 cm. Find the nature and the power of lens to be used for seeing clearly the object at infinity.

### Solution:

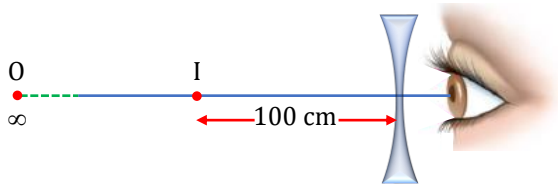
Given, u = –∞; v = –100 cm

From lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{100}+\frac{1}{\infty}=\frac{1}{f}$$

$$f=-100\ \text{cm}$$

P = –1D (Diverging lens)




### Illustration 109:

Far point of a person is 50 cm. Find power of spectacles required for him to see an object placed at 200 cm.

### Solution:

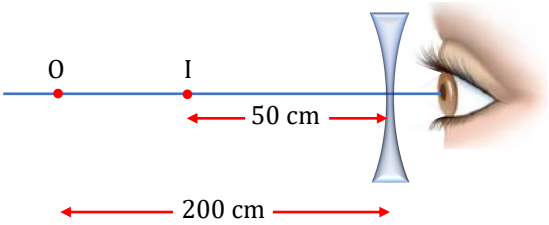
Given, u = –200 cm; v = –50 cm

From lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{50}+\frac{1}{200}=\frac{1}{f}$$

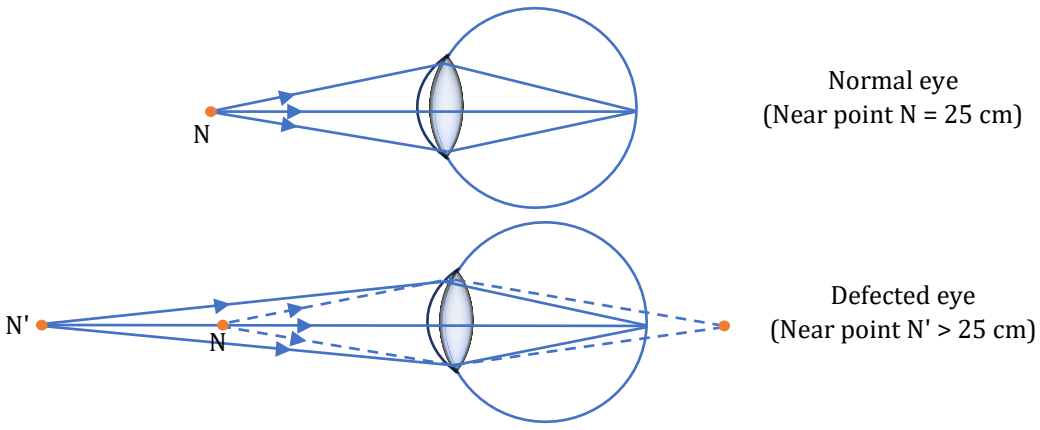
$$f=-\frac{200}{3}$$

$$P=\frac{1}{f}=-\frac{100\times 3}{200}=-1.5D$$




### Hypermetropia (Far-sightedness)

- In this defect distant objects are clearly visible but nearby object are not clearly visible.
- Near point of eye shifted away from the eye.




This blurred effect is due to incoming light being focused behind, instead of on the retina wall due to insufficient accommodation by the lens.

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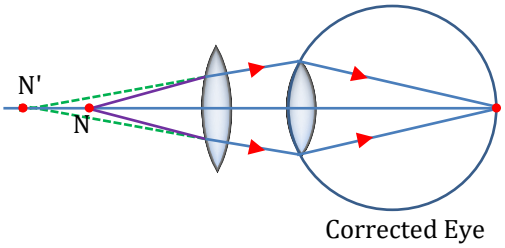
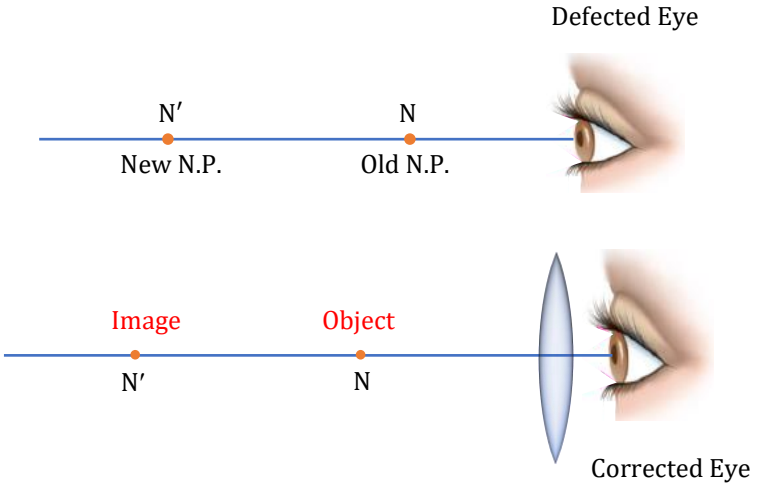
### Correction of defect

To remove this defect **converging lens (convex lens)** is used.




### To solve the problems




$u = -N, v = -N'$

from lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-N'}-\frac{1}{-N}=\frac{1}{f} \Rightarrow \boxed{P=\frac{1}{f}=\frac{1}{N}-\frac{1}{N'}}$$

### Illustration 110:

A person can see objects placed beyond distance of 1 m. Find the optical power of the spectacles compensating the defect of vision for this eye.

### Solution:

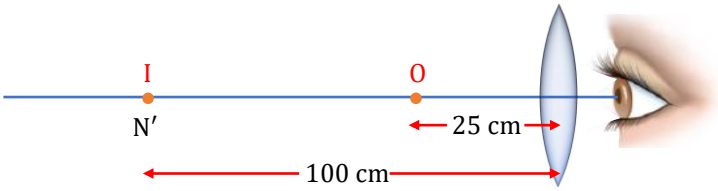
Given, $u = -25$ cm; $v = -100$ cm

From lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow -\frac{1}{100}+\frac{1}{25}=\frac{1}{f}$$

$$f=\frac{100}{3}\text{ cm} \qquad \text{(convex lens)}$$

$$P=\frac{1}{f} \Rightarrow P=\frac{100\times 3}{100}=3\text{D}$$



### Illustration 111:

A far-sighted person can see object beyond 49 cm clearly if distance between corrective lens and eye lens is 1 cm, then find focal length of corrective lens?

### Solution:

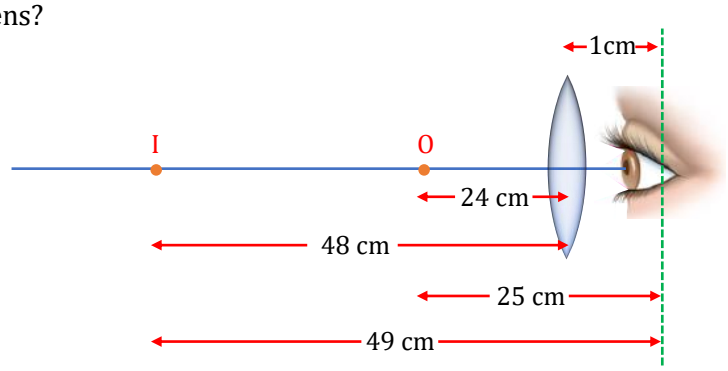
Given, u = –24 cm; v = –48 cm

From lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$-\frac{1}{48}+\frac{1}{24}=\frac{1}{f}$$

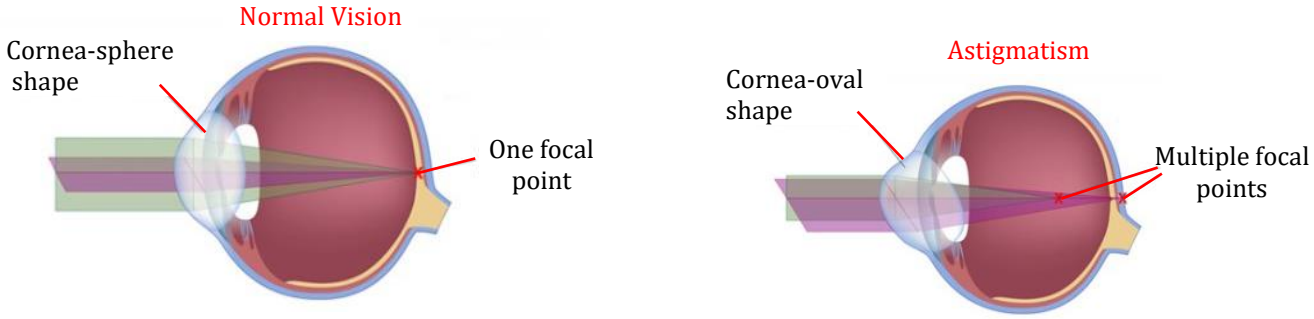
$f = 48$ cm




### Presbyopia

In this case, both nearby and distant objects are not clearly visible. To remove this defect, two separate lenses one for myopia and other for hypermetropia are used or **bifocal lenses** are used.

### Astigmatism




This defect arises due to the fact that the cornea is not perfectly spherical and has different curvatures for horizontally and vertically lying objects. Hence, objects in one direction are well focused whereas objects in the perpendicular direction are not well focused. This defect can be corrected by using **cylindrical lenses**. The cylindrical lenses are designed in such a way so as to compensate for the irregularities in the curvature of cornea.

## BEGINNER'S BOX-11

**1.** What should be the focal length of the reading spectacles for a person for whom near point is 50 cm ?

**2.** A person can see objects placed beyond distance of 1 m. Find the optical power of the spectacles compensating the defect of vision for this eye.

**3.** A near sighted man can see objects clearly up to a distance of 1·5m. Calculate the power of the lens of the spectacles necessary for the remedy of this defect.

## Natural Phenomena of Light

### The Rainbow

The rainbow is an example of the dispersion of sunlight by the water drops in the atmosphere. This is a phenomenon due to combined effect of dispersion, refraction and reflection of sunlight by spherical water droplets of rain. The conditions for observing a rainbow are that the Sun should be shining in one part of the sky (say near western horizon) while it is raining in the opposite part of the sky (say eastern horizon). An observer can therefore see a rainbow only when his back is towards the Sun.

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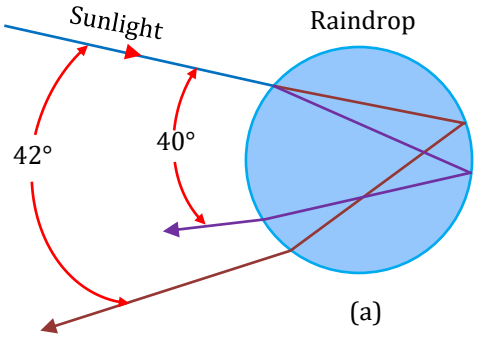
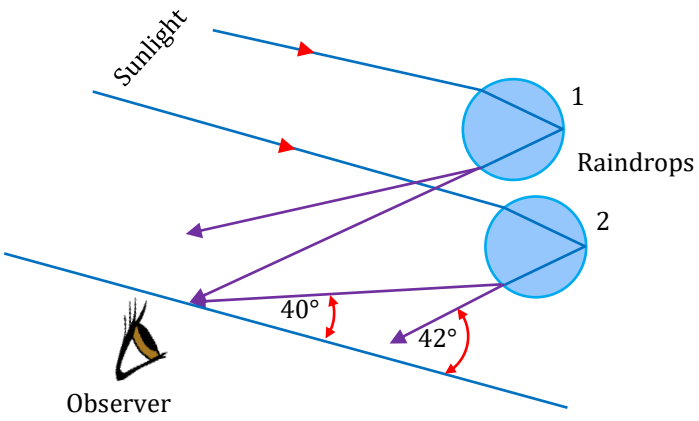
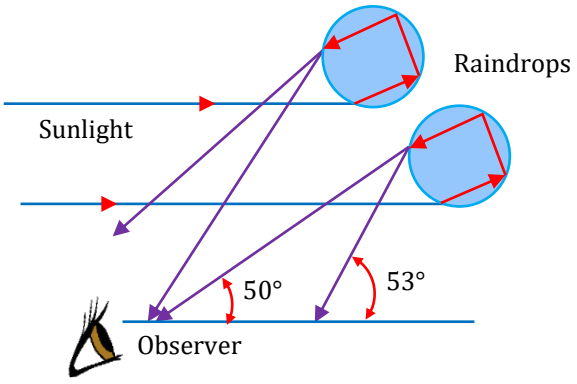
In order to understand the formation of rainbows, consider (a). Sunlight is first refracted as it enters a raindrop, which causes the different wavelengths (colours) of white light to separate. Longer Wang length of light (red) are bent the least while the shorter wavelength (violet) are bent the most. Next, these component rays strike the inner surface of the water drop and get internally reflected if the angle between the refracted ray and normal to the drop surface is greater than the critical angle (48° in this case). The reflected light is refracted again when it comes out of the drop, as shown in the figure. It is found that the violet light emerges at an angle of 40° related to the incoming sunlight and red light emerges at an angle of 42°. For other colours, angles lie in between these two values. Figure (b) explains the formation of primary rainbow. We see that red light from drop 1 and violet light from drop 2 reach the observer's eye. The violet from drop 1 and red light from drop 2 are directed at level above or below the observer. Thus, the observer sees a rainbow with red colour on the top and violet on the bottom. The primary rainbow is a result of three-step process, that is, refraction, reflection and refraction.




(a)




(a) Primary rainbow




(b) Secondary rainbow

Rainbow: (a) The sun rays incident on a water drop get refracted twice and reflected internally by a drop; (b) Enlarged view of internal reflection and refraction of a ray of light inside a drop forming primary rainbow; and (c) secondary rainbow is formed by rays undergoing internal reflection twice inside the drop. When light rays undergoes two internal reflections inside a raindrop, instead of one as in the primary rainbow, a secondary rainbow is formed as shown in (c). It is due to four-step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. Further, the order of the colours is reversed in it as is clear from (c).

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## Scattering of light

The deflection of light energy by fine particles of solid, liquid or gaseous matter from the main direction of the beam is called the scattering of light.

The basic process involved in scattering is the absorption of light by the molecules followed by its re-radiation in different directions. The intensity of the scattered light depends on:

(i) the wavelength ($\lambda$) of light (ii) the size of the particles causing scattering.

Depending upon the size of the scatterers, the following two situations arise:

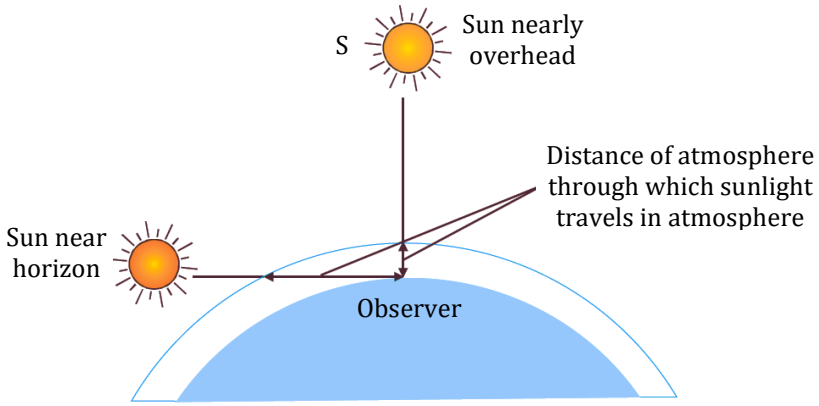
**(a)** If the scattering particles (air molecules) are of size smaller than the wavelength of light, the intensity of the scattered light (I) varies inversely as the fourth power of the wavelength of light, This statement, which holds for elastic scattering, is known as the Rayleigh's law of scattering. Since the wavelength of blue light is less than that of red light, blue light is scattered most while the red light least. Blue colour of sky, reddish appearance of Sun during sunrise and sunset are due to this phenomenon as discussed below.

Further, it is due to this reason that red signals are used to indicate danger. Such signals go to large distances without an appreciable loss due to scattering.

**(b)** If the scattering particles are of sizes greater than the wavelength of light (e.g., dust particles, water droplets), Rayleigh's law of scattering is not applicable, and all colours are scattered equally. It is due to this reason that clouds generally appear white.

**Blue colour of the sky**

If an observer (O) looks at the sky when the Sun is overhead at noon as shown by position S in figure it is the scattered light that is received by the observer. Since blue light is scattered more (almost six times) than red, the sky appears blue to the observer.




## Reddish appearance of the Sun during sunrise and sunset

At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere (Figure). Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes; therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.

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## BEGINNER'S BOX — ANSWERS KEY

### BEGINNER'S BOX-1

**1.** 0.9 m **2.** 4 **3.** 60 m **4.** 60° **5.** 6.18m

**6.** 3

### BEGINNER'S BOX-2

**1.** (A)P (B)R,S (C)S (D)Q,S **2.** u = –0.1 m

**3.** $\frac{f}{3}$, m = $\frac{2}{3}$ so image will be virtual, erect and smaller than the object

**4.** 19.35 cm behind the mirror. **5.** u = –12 cm **6.** – 40 cm

**7.** – 54 cm, $h_I$ = –5 cm, m = –2 real, inverted, magnified

### BEGINNER'S BOX-3

**1.** 30 cm **2.** $2\cos^{-1}\left(\frac{\mu}{2}\right)$

**3.** Frequency is a characteristic of source, so it will not change with change in medium.

### BEGINNER'S BOX-4

**1.** $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$ **2.** $C=\sin^{-1}\frac{1}{\sqrt{2}}=45^0$ **3.** i < 45°

### BEGINNER'S BOX-5

**1.** 3.63 cm **2.** –3.07 cm

### BEGINNER'S BOX-6

**1.** 15 cm, real, inverted, small in size **2.** + 15 cm **3.** 1.66

**4.** 7.5 D **5.** 7.5 D **6.** 2 **7.** – f (1+n)

### BEGINNER'S BOX-7

**1.** 20.5 cm **2.** $f_1$ = –45 cm and $f_2$ = 30 cm **3.** 1.6 cm.

**4.** $f_1$ = –20 cm, $f_2$ = +15 cm **5.** 0.2 cm

### BEGINNER'S BOX-8

**1.** 30° **2.** $\mu > \frac{2}{\sqrt{3}}$ **3.** $\sqrt{3}$ **4.** 90°

**5.** $\sqrt{2}$ **6.** 70°

### BEGINNER'S BOX-9

**1.** 0·090° **2.** (a) 6.59°, 6.41° (b) 0.18° (c) 0.0276 **3.** 0.2

### BEGINNER'S BOX-10

**1.** 5 **2.** 204.17 cm **3.** 6, 25 cm **4.** 24°

**5.** (i) –12, 65 cm (ii) –14.4, 64.17 cm

### BEGINNER'S BOX-11

**1.** + 50 cm **2.** +3 D **3.** – 0.66 D

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